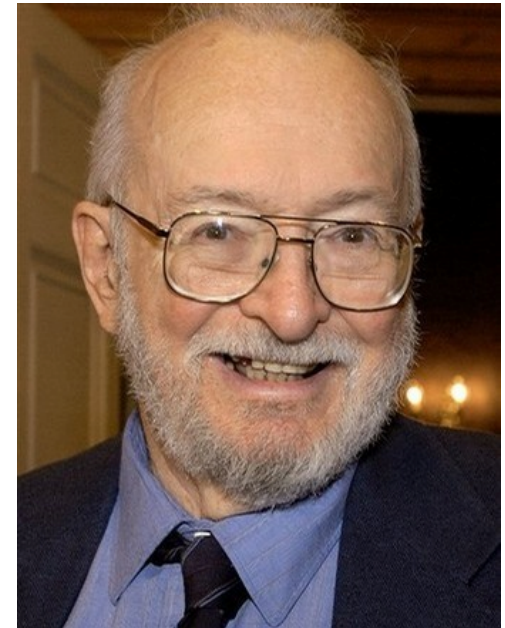


Chapter 13

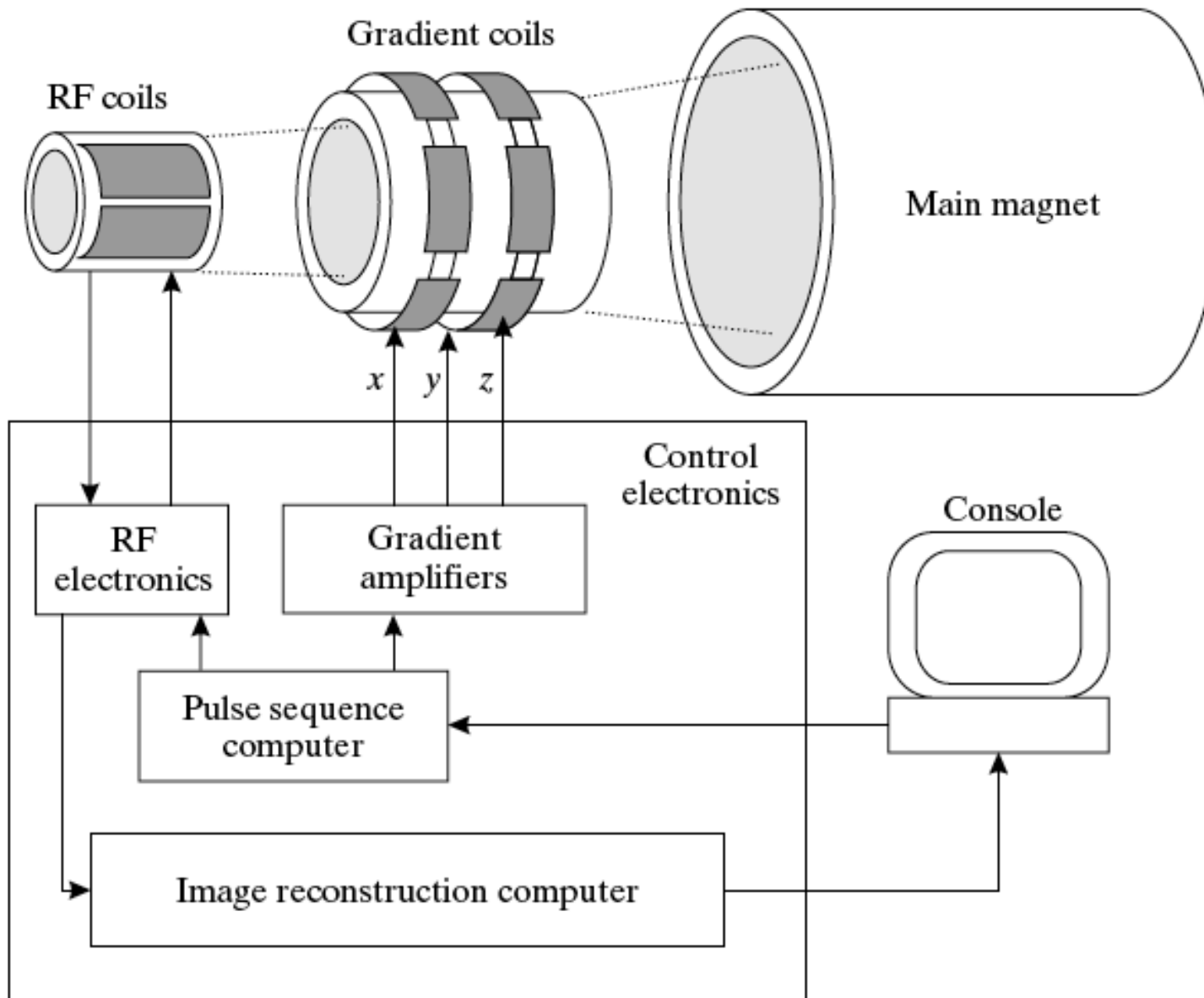
Magnetic Resonance Imaging

- Paul Lauterbur conceived of spatially encoding NMR in the early 1970s (he worked at Pitt), making it a *tomographic* imaging modality. (Nobel Prize 2003)



- Patients objected to “Nuclear” because it sounds dangerous, so it was changed to “Magnetic Resonance Imaging” (MRI), or simply “Magnetic Resonance” (MR).

Principle components of an MRI scanner



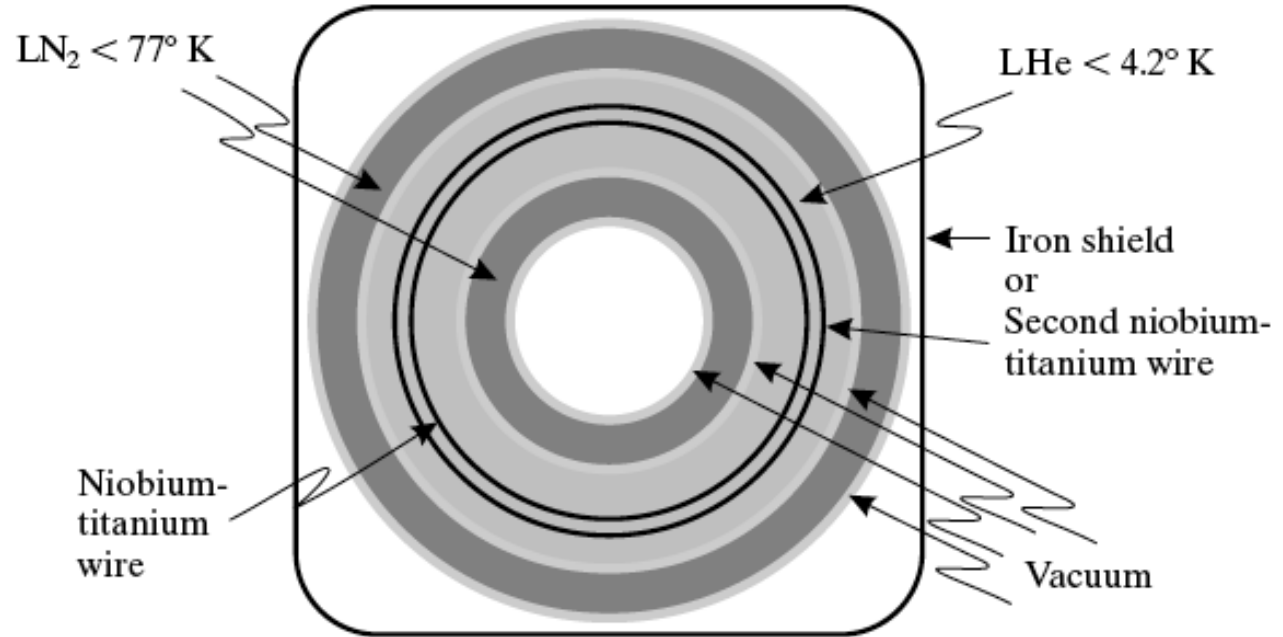
Room must be copper lined (Faraday cage) to eliminate RF noise.

Steel objects must be kept away from main magnet.

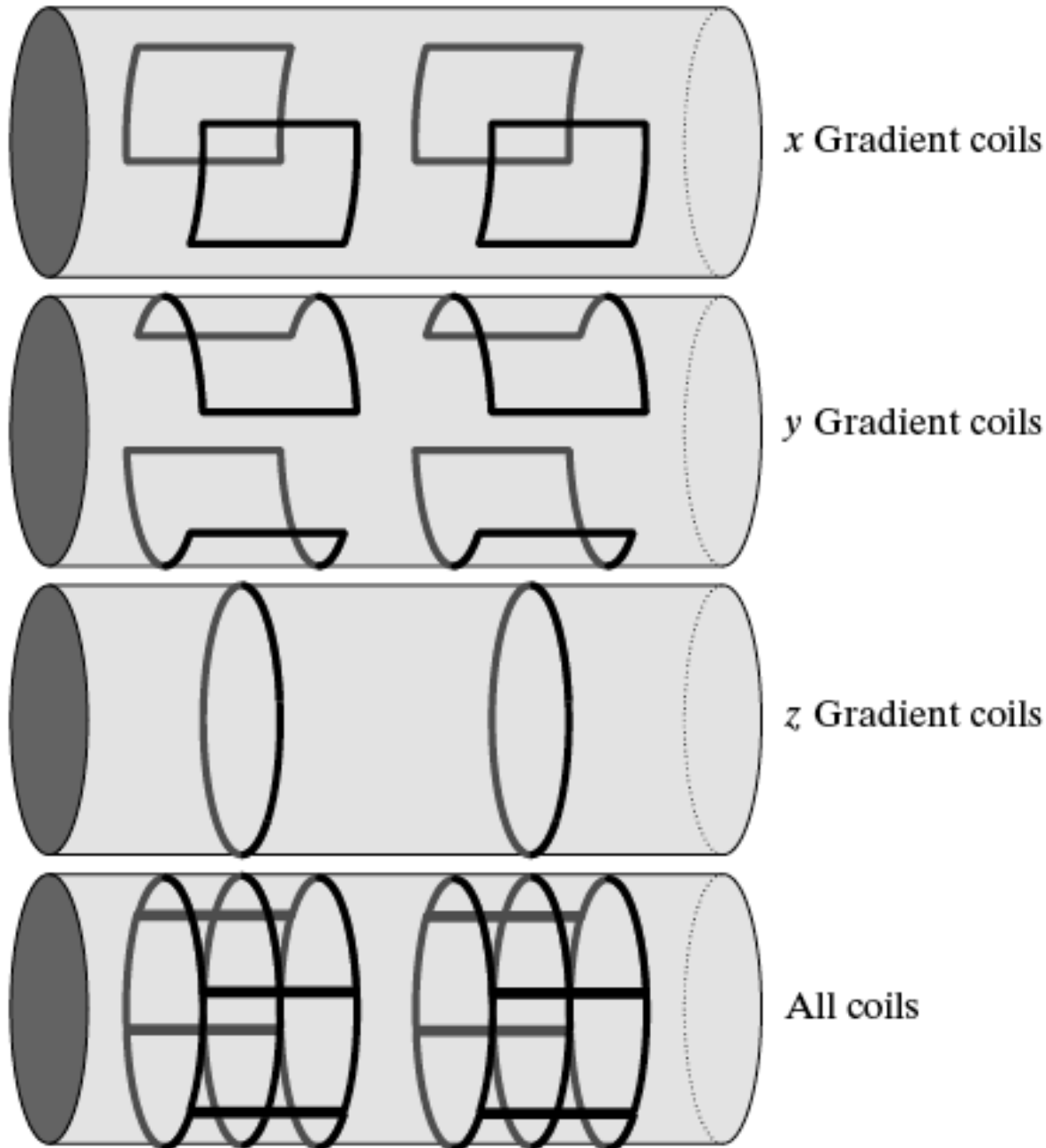
“Mu-metal” can be used to shield some magnetic fields.

Magnet

- Generally a long narrow cylinder, though some scanners are “open” magnets with two poles, giving access for surgery.
- Superconducting magnet, supercooled, 2.8 million joules.
- 0.5 T to 9 T, with 1.5 T most common, and 3 T growing.
- Homogeneous to 5 ppm, with *active shimming* (small coils).
- Fringe fields reduced by second coils outside the primary.



Three sets of *gradient coils*



Small changes to the strength of the main magnet's field \mathbf{B}_0 , but not its direction, as a function of x , y , and z .

These gradient coils carry large currents (100-200 amperes) and are thus subject to large *Lorentz forces*.

They can be switched on and off very quickly, *slew rate* limited by peripheral nerve stimulation, noisy!

Gradients in the magnitude of ***B***

- Magnetic field still oriented in the z direction, just weaker or stronger depending on location.

$$\text{vector} \longrightarrow B = \underbrace{(B_0 + G_x x + G_y y + G_z z)}_{\text{magnitude}} \hat{z}$$

- The total gradient can be written as a vector, whose magnitude has units of Gauss per centimeter.

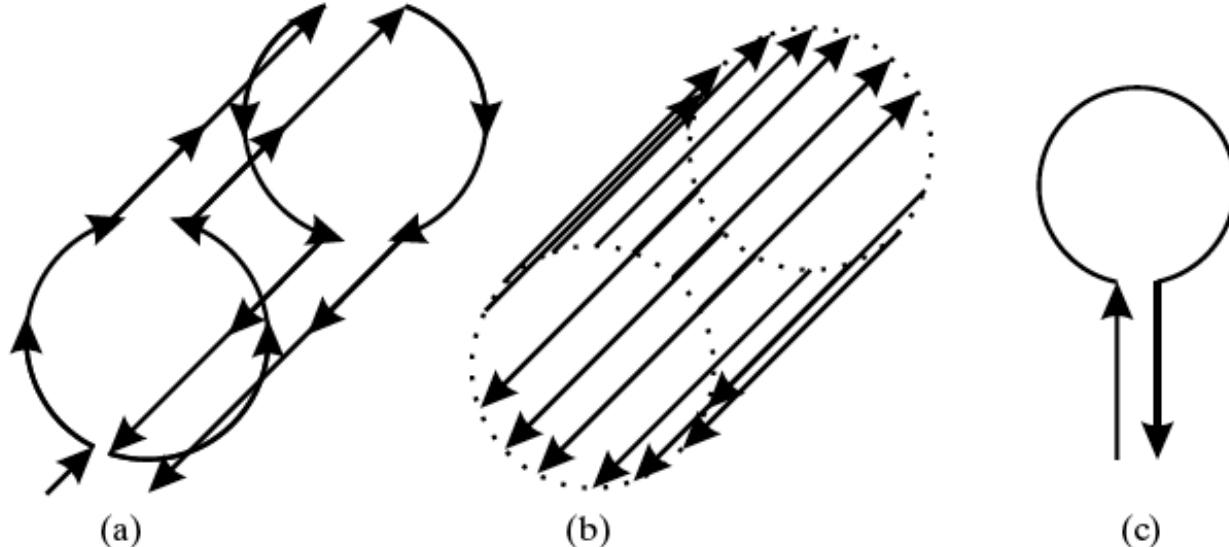
$$G = (G_x, G_y, G_z)$$

- The dot product with location $r = (x, y, z)$ then yields Gauss added or subtracted to the strength of the big magnet at a particular location.

$$B = (B_0 + G \cdot r) \hat{z}$$

Radio-Frequency (RF) coils

- RF coils (*resonators*) serve to both *transmit* (tip the longitudinal magnetization into the transverse plane) and *receive* (the signals produced by the precessing transverse magnetization).
- Two basic types
 - *Volume coils* surround the object and have good uniformity of field, thus guaranteeing the same tip angle for all samples, and avoiding variation in pixel intensity (a, b, below)
 - *Surface coils* are placed close to the object and have high sensitivity; they can be made somewhat more uniform by using arrays (c, below).



Frequency encoding of spatial position

- The Larmor frequency produced by the precession of the transverse magnetization in magnetic field \mathbf{B} is $\nu = \gamma B$
- Adding the gradients makes the frequency depend on location

$$\nu(\mathbf{r}) = \gamma(B_0 + \mathbf{G} \cdot \mathbf{r})$$

- We can excite a particular axial slice using the z-gradient to make Larmor frequency a function of z.

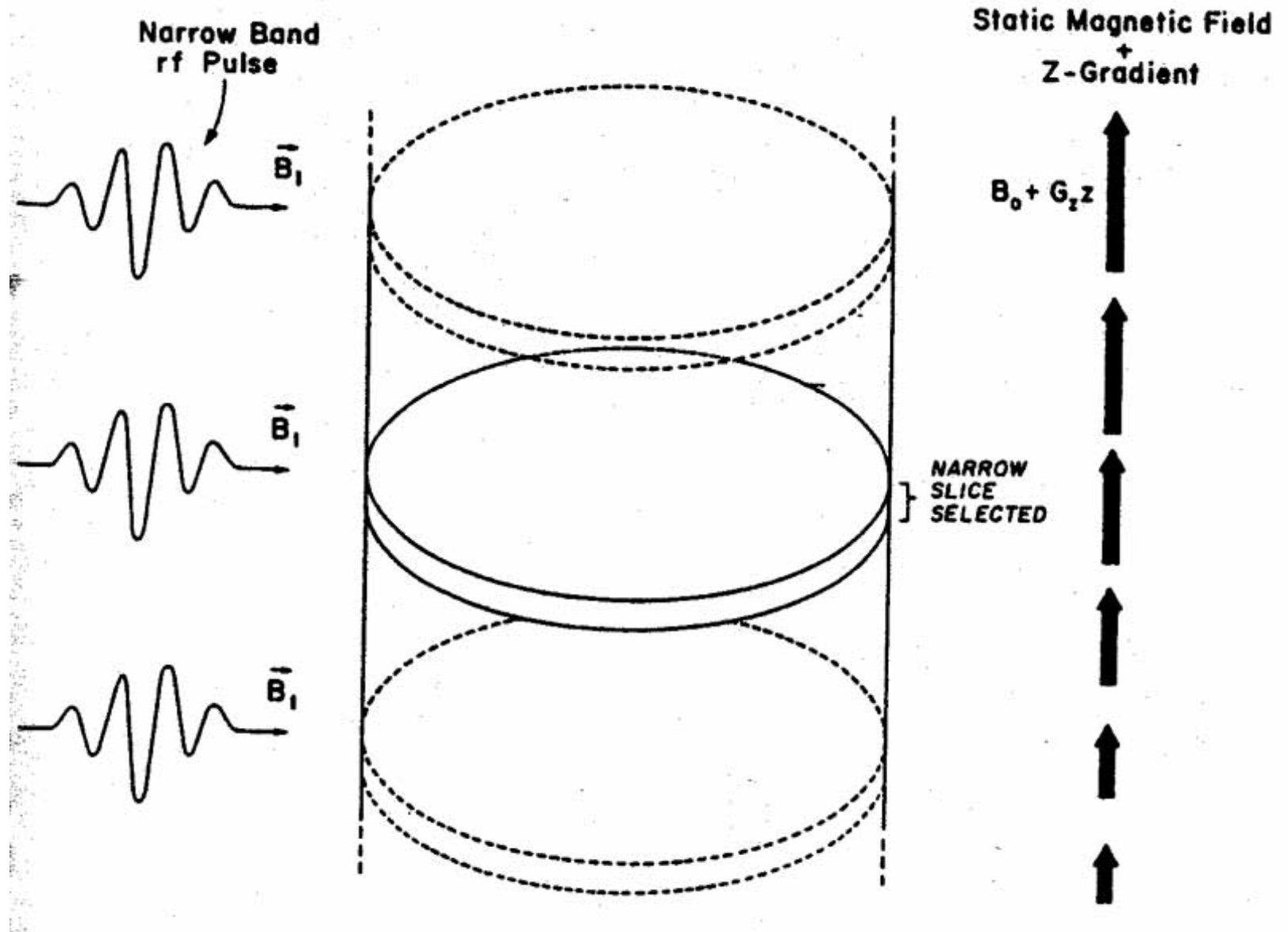
$$\mathbf{G} = (0, 0, G_z)$$

$$\begin{aligned}\nu(\mathbf{r}) &= \gamma(B_0 + \mathbf{G} \cdot \mathbf{r}), \\ &= \gamma(B_0 + G_z z) .\end{aligned}$$

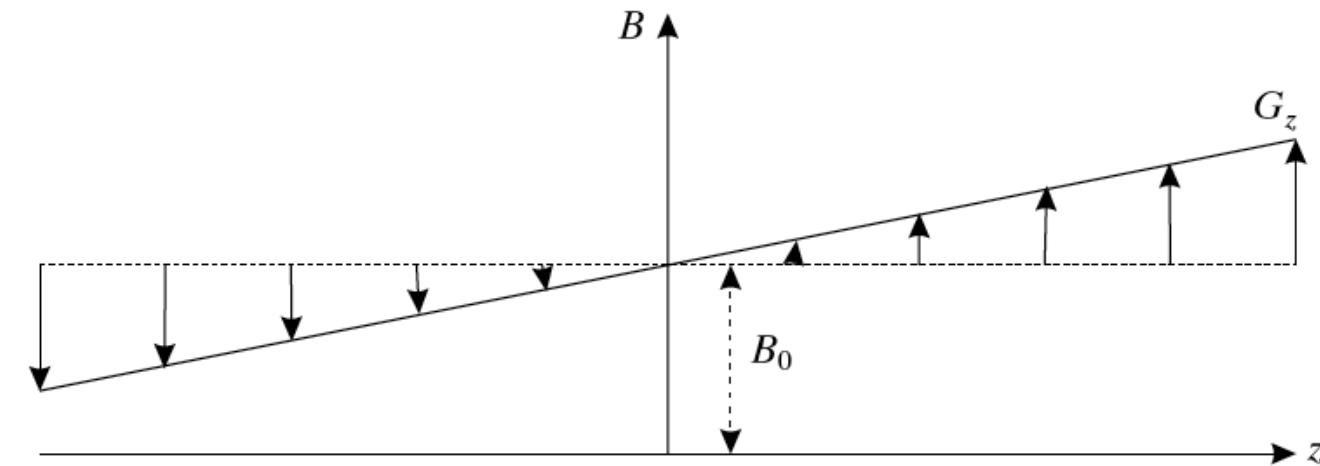
How to localize excitation:

Slice selection by z gradient during RF pulse containing a narrow band of frequencies

Fourier Transform of sinc pulse is rect function, a band of frequencies that corresponds to a slice in the z-direction with the correct B (not B_0).



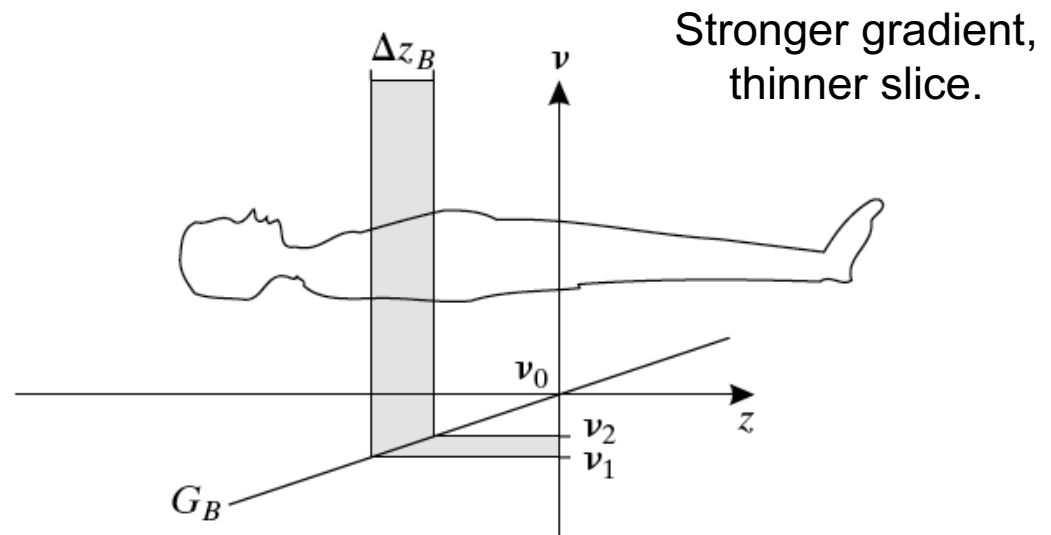
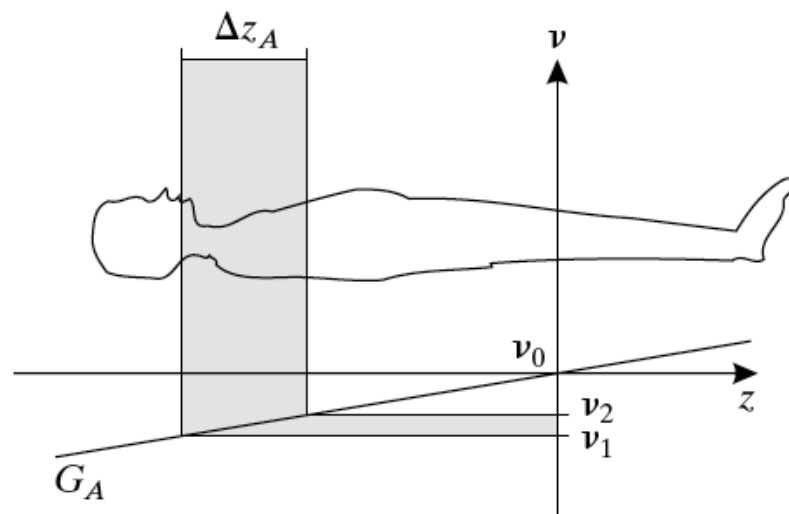
- Thickness and location of slice determined by strength of gradient and particular frequency band of RF pulse.



$$\bar{\nu} = \frac{\nu_1 + \nu_2}{2}$$

$$\Delta\nu = |\nu_2 - \nu_1|$$

→ → → → →
Size and orientation of main field

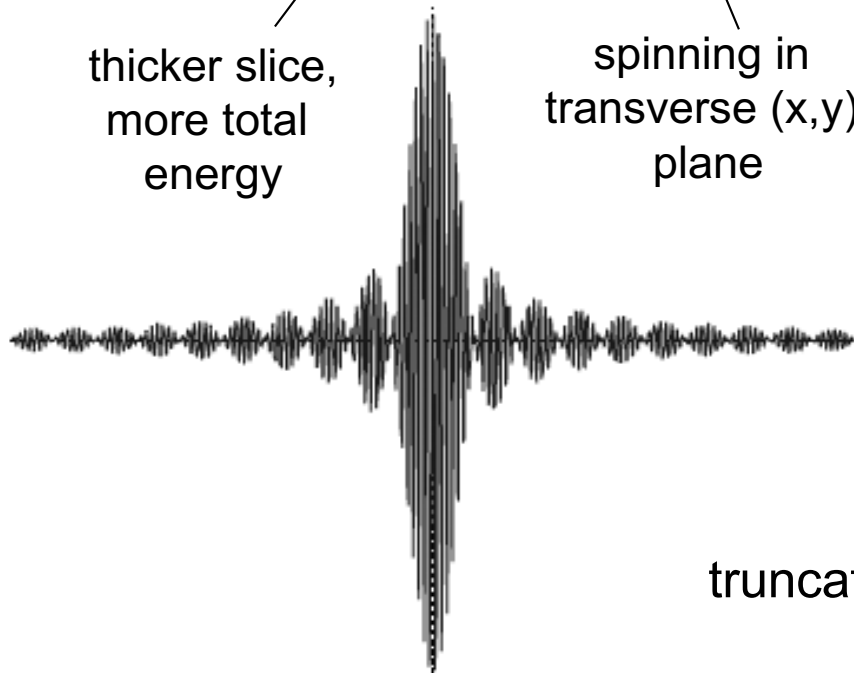


RF pulse,

$$s(t) = A\Delta\nu \operatorname{sinc}(\Delta\nu t) e^{j2\pi\bar{\nu}t}$$

thicker slice,
more total
energy

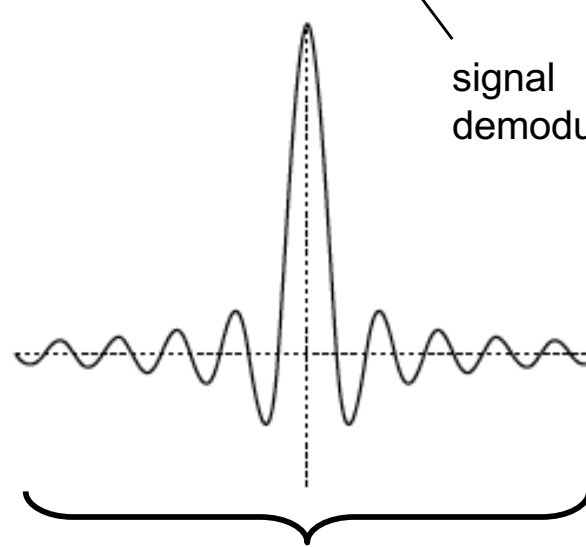
spinning in
transverse (x,y)
plane



its envelope and Fourier transform

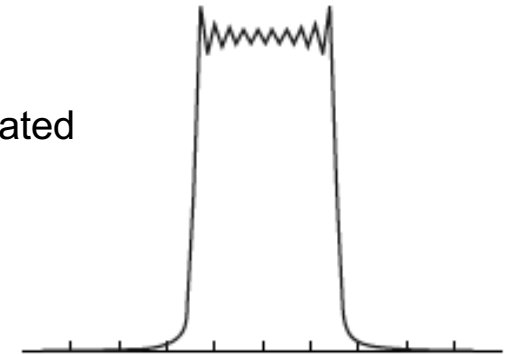
$$B_1^e(t) = s(t)e^{-j2\pi\bar{\nu}t} \quad S(\nu) = A \operatorname{rect}\left(\frac{\nu - \bar{\nu}}{\Delta\nu}\right)$$

signal
demodulated



truncation to duration τ_p results in “ringing”

slice in
frequency
domain



The resulting tip angle,

$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t) dt$$

whose Fourier transform is...

$$\alpha(z) = \gamma A \operatorname{rect}\left(\frac{z - \bar{z}}{\Delta z}\right)$$

...constant across a slice

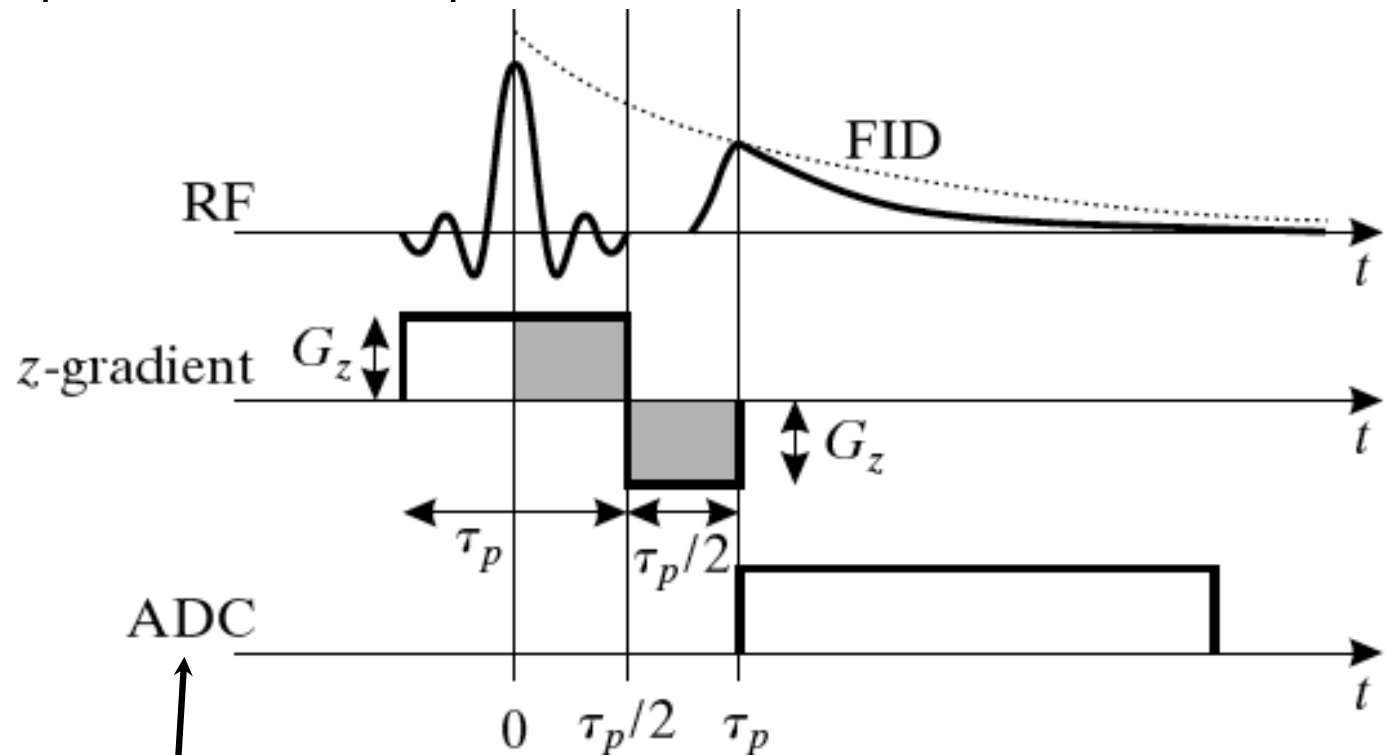
Refocusing Gradients

- During the RF excitation with the z-gradient activated, spins across the slice go at slightly different frequencies, and will thus accumulate phase differences depending on z , of

$$\underbrace{\gamma G_z(z - \bar{z})}_{\text{Larmor freq}} \times \underbrace{\tau_p/2}_{\text{pulse duration}} = \text{phase}$$

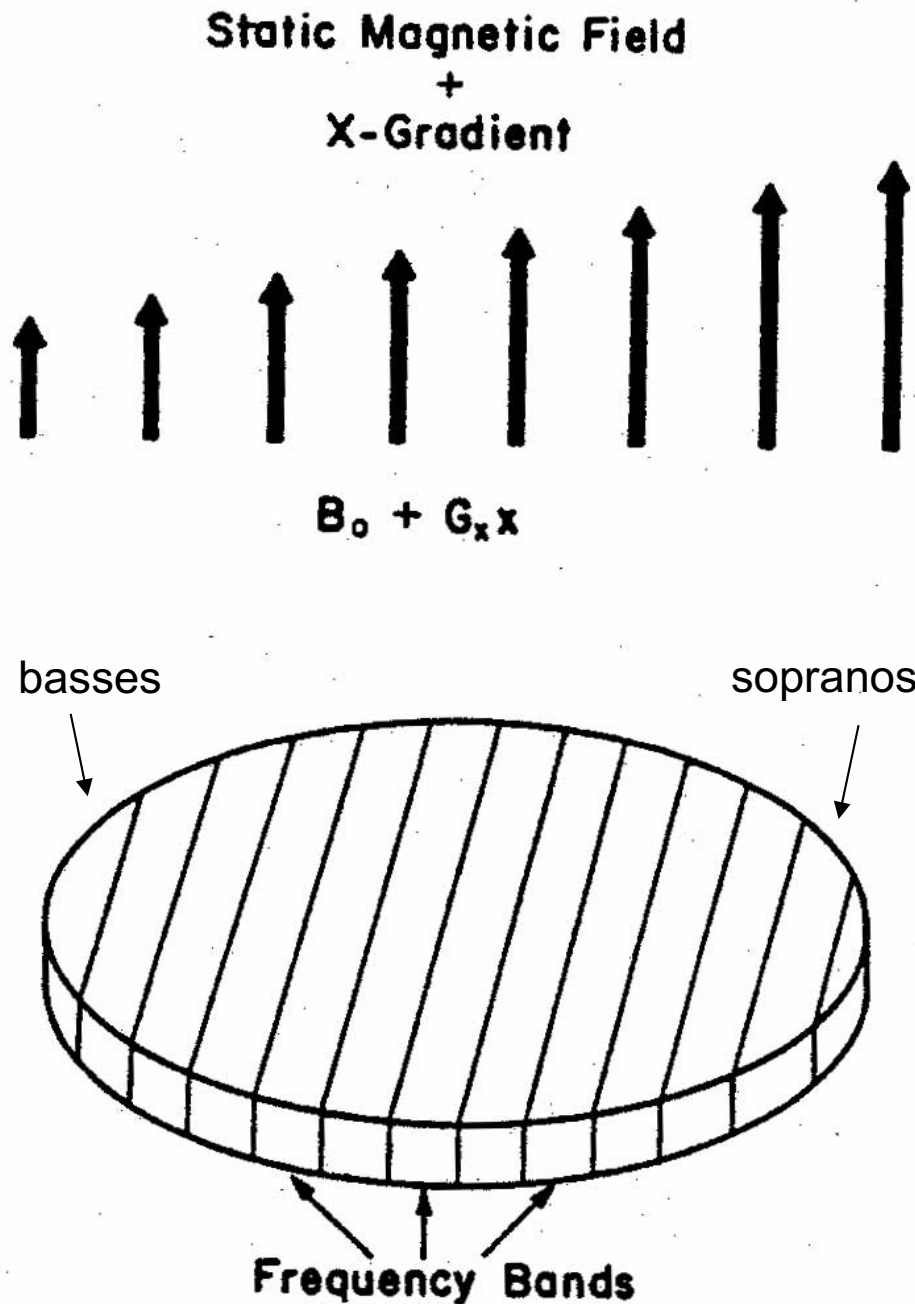
← divided by 2 because pulse “occurs” half-way through τ_p establishing where $t = 0$.

Pulse sequence uses second period of $\tau_p/2$ with G_z turned on in the other direction bring these phases back in line, for the *free induction decay (FID)*.



analog to digital conversion to read out the FID

Frequency Encoding with Readout Gradients

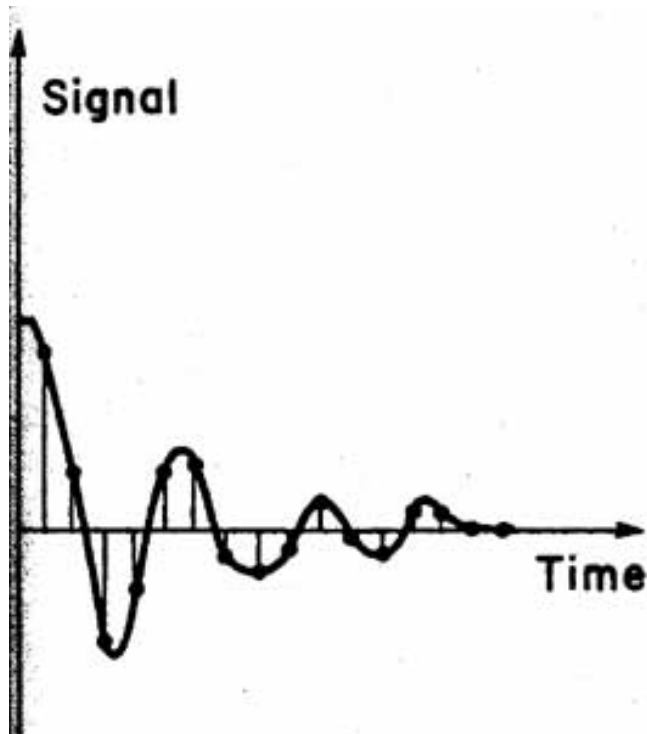


We now have a single slice energized and generating RF from its precessing transverse magnetization.

We have one dimension localized. How do we get the second dimension?

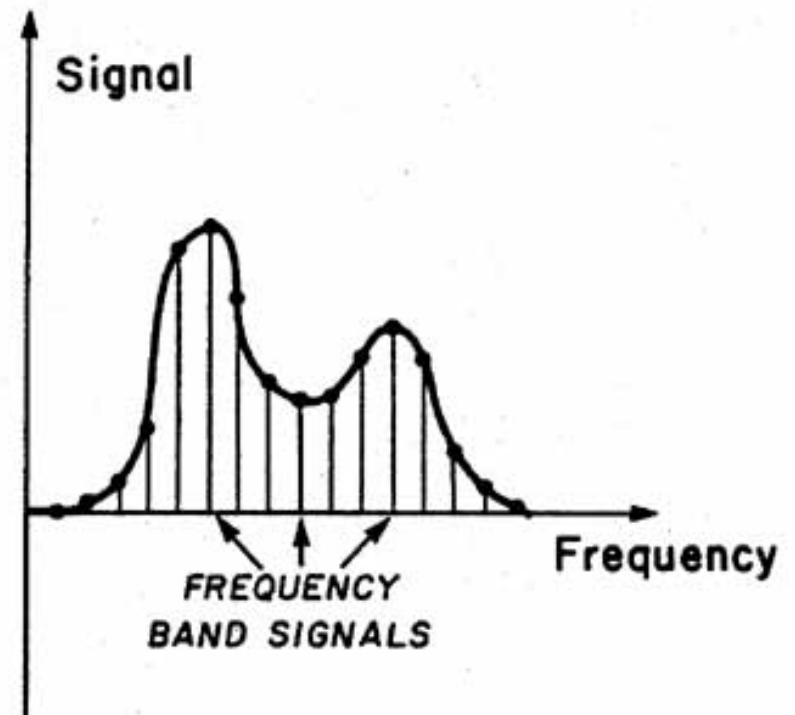
Within each slice, different bands of RF are created across the slice by a gradient G_x during the readout.

Bands within the slice are separated using the Fourier Transform.



Free Induction Decay

One-Dimensional
Fourier Transform



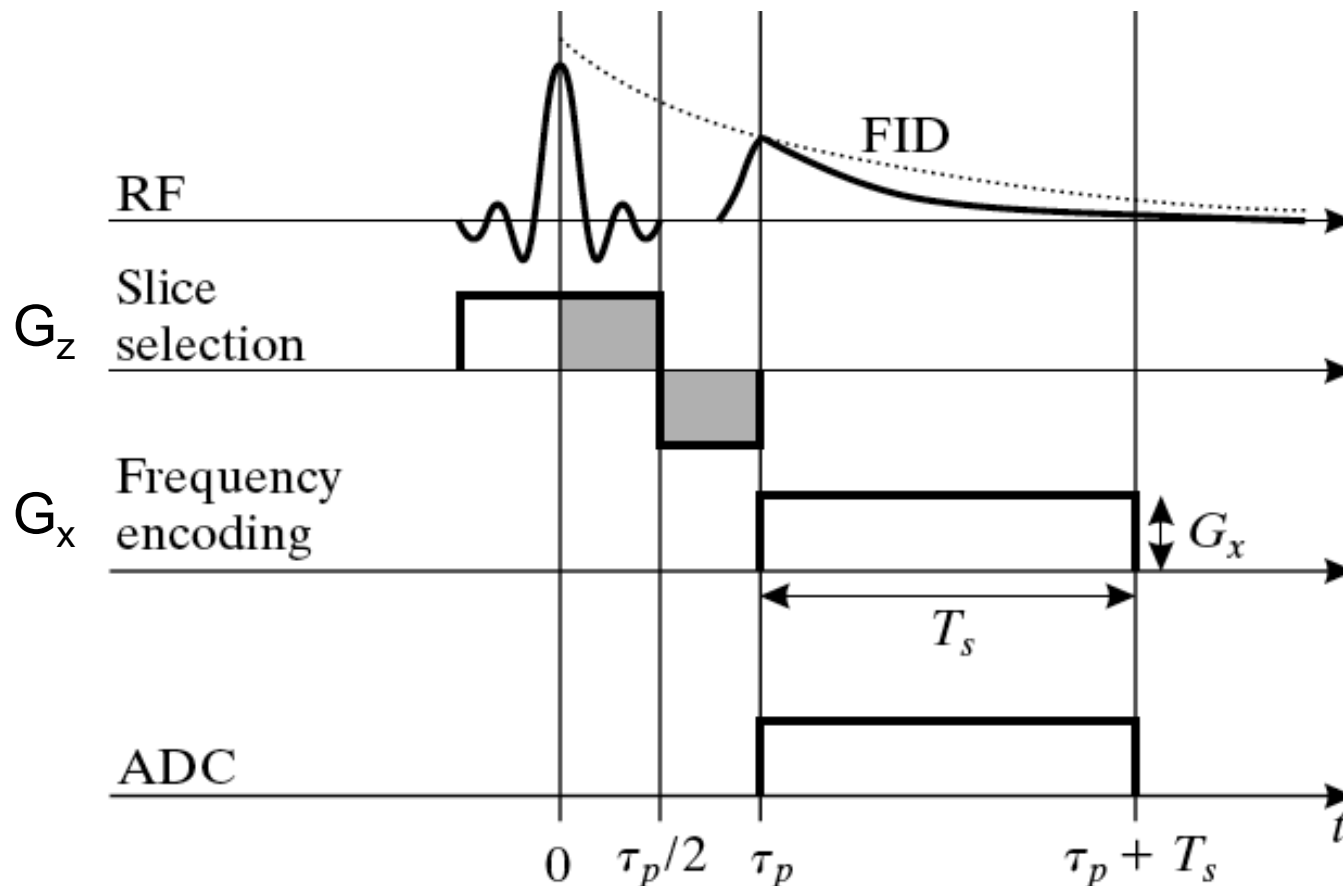
Different frequencies,
depending on location in
the x direction

Frequency Encoding with Readout Gradients

- The x-gradient causes the Larmor frequency to vary across the slice

$$\nu(x) = \gamma(B_0 + G_x x)$$

during the sampling interval T_s , allowing bands in the slice to be distinguished.



- The image we are seeking is the *effective spin density* $f(x,y)$

combined gain constant transverse magnetization just after α pulse transverse relaxation due to dephasing

$$f(x, y) = \underbrace{A}_{\text{combined gain constant}} \underbrace{M_{xy}(x, y; 0^+)}_{\text{transverse magnetization just after } \alpha \text{ pulse}} \underbrace{e^{-t/T_2(x,y)}}_{\text{transverse relaxation due to dephasing}}$$

Note: Prince leaves out the “xy” in Eq. 13.22

- The actual received signal $s(t)$, without any read gradients

(-) from original diff. eq. solution Eq. 12.8

not the RF pulse $s(t)$ on slide 10 \rightarrow $s(t) = \underbrace{e^{-j2\pi\nu_0 t}}_{\text{all spinning at } \nu_0 \text{ in transverse plane}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$

- MR hardware *demodulates* the signal to *baseband signal* $s_0(t)$

Demodulation, *also called Superheterodyning* in radio receivers, puts us in the rotating frame of reference, by multiplying with a phasor that rotates with that frame.

exactly cancels (-) phasor in $s(t)$, shifting it to the baseband.

$$s_0(t) = \underbrace{e^{+j2\pi\nu_0 t}}_{\text{exactly cancels (-) phasor in } s(t)} s(t)$$

Without gradients $s_0(t)$ is just DC,, the average value of the slice.

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$$

Frequency encoding is decoded by the Fourier transform

With the x-gradient on while reading...


$$\nu(x) = \gamma(B_0 + G_x x)$$

the difference
from ν_0 at (x,y)
due to G_x

The demodulated
(baseband) signal is...

$$s_0(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \overbrace{e^{-j2\pi\gamma G_x x t}}^{\text{the difference from } \nu_0 \text{ at } (x,y) \text{ due to } G_x} dx dy$$

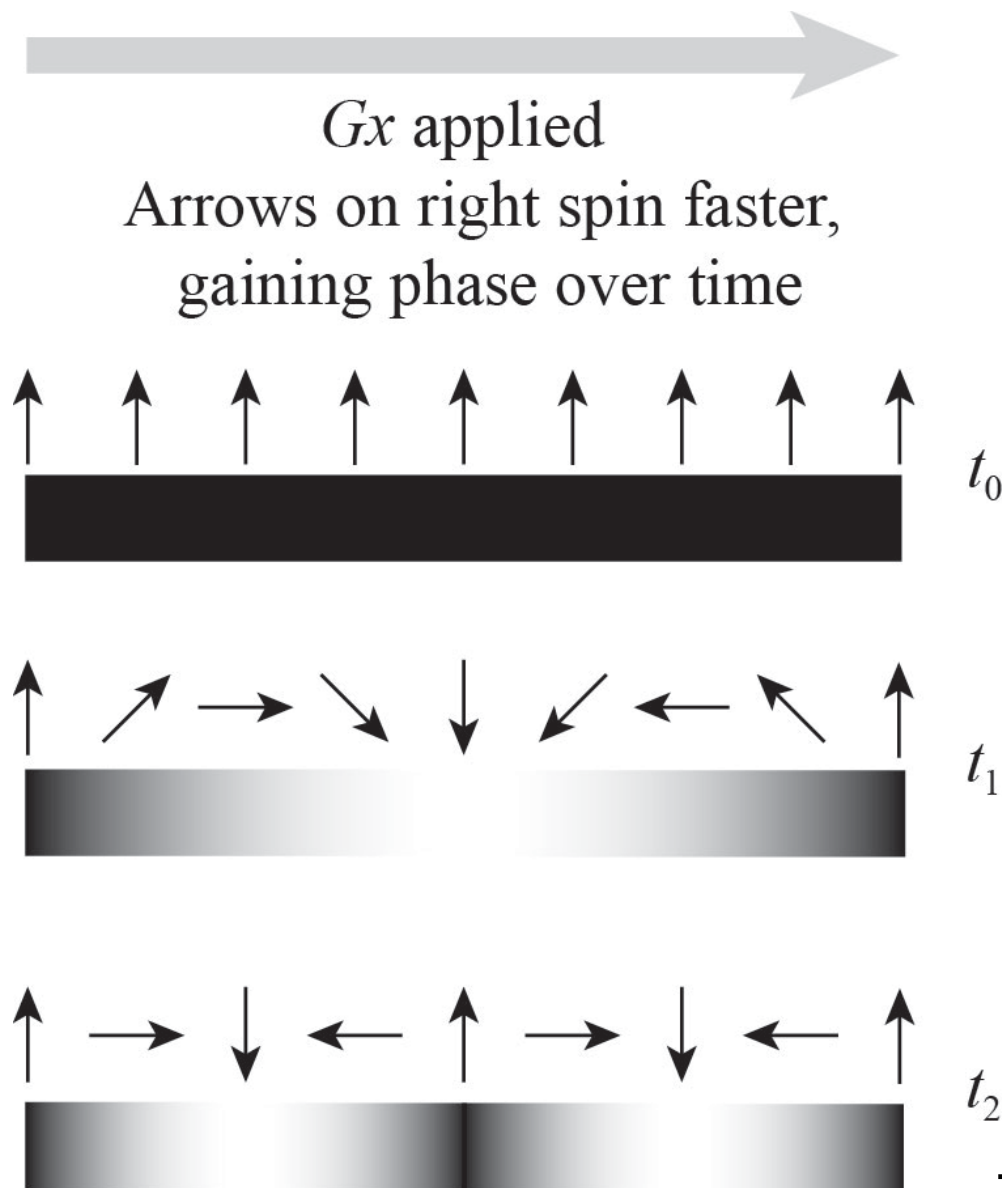
This is the classic form of a 2D Fourier Transform of $f(x,y)$...

$$F(u, 0) = s_0 \left(\frac{u}{\gamma G_x} \right)$$


not of *time*, but of *distance*,
where spatial frequencies u and v are

$$u = \gamma G_x t \qquad v = 0$$

spatial frequency u
increases with time

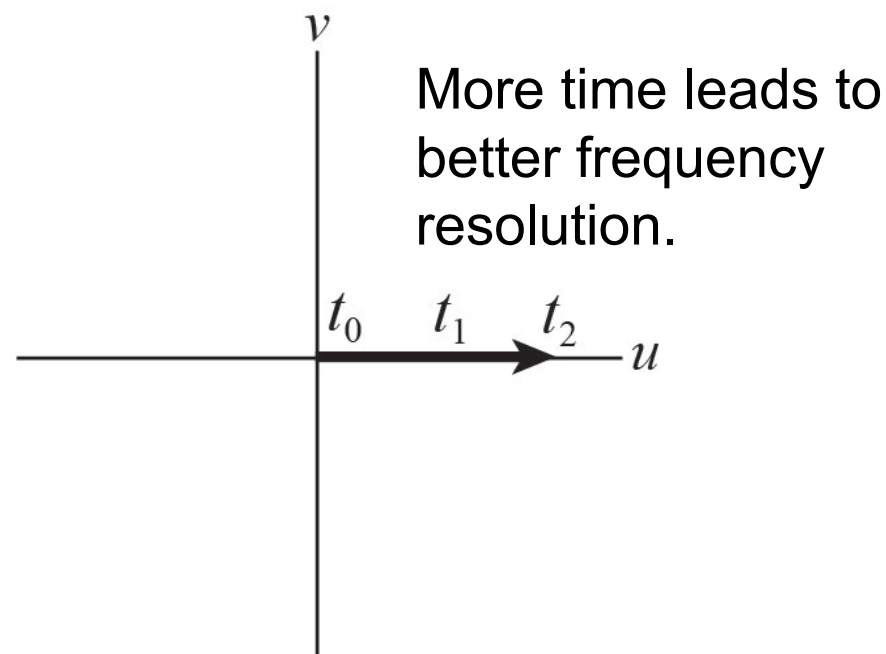


Arrows are $M_{x'y'}$ (in rotating frame)

The magnitude of each arrow is the signal strength from that voxel.

k is spatial frequency

k -space traversed over time as spatial frequency u increases



The total signal at a given time is a particular spatial Fourier component.

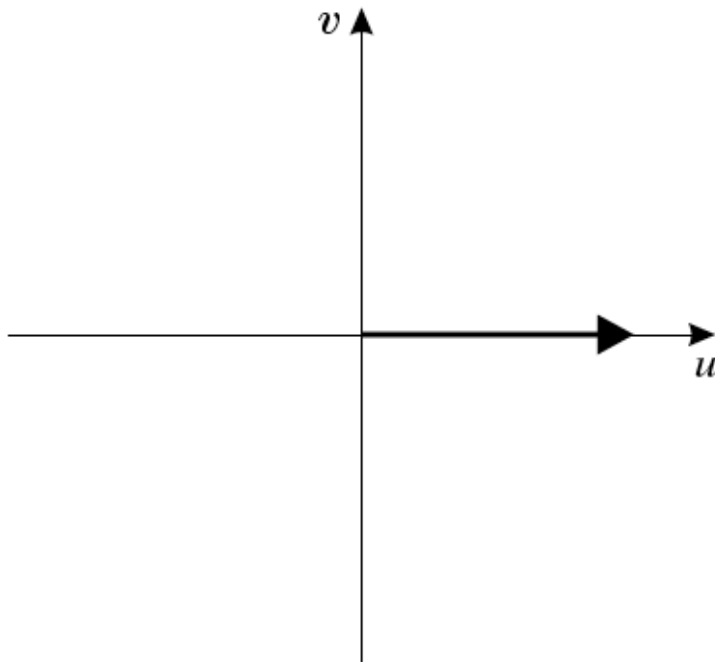
The temporal signal is complex with the y axis in space imaginary.

k-space

- Arises from the convention in physics where wave number k represents a spatial frequency.
- Prince uses u and v instead of the traditional k_x and k_y

$$k_x = u \quad k_y = v$$

- Pulse sequences become a *Fourier trajectory* in *k-space*.



The single scan across the slice in the x direction just described is represented by this *Fourier trajectory*.

It is a trajectory in the sense that with passing time higher spatial frequencies are represented (larger phase angle for a given x).

$$\nearrow u = \gamma G_x t \nwarrow$$

spatial frequency is equivalent to temporal phase

If we can cover all of Fourier space, we can recover $f(x,y)$

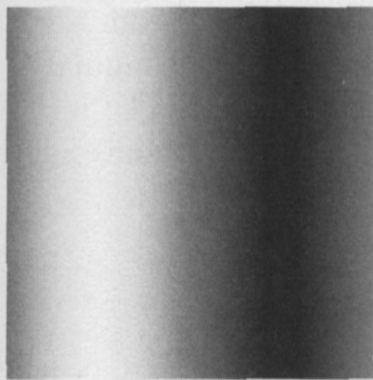
Recall Definition of Fourier in 2D

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

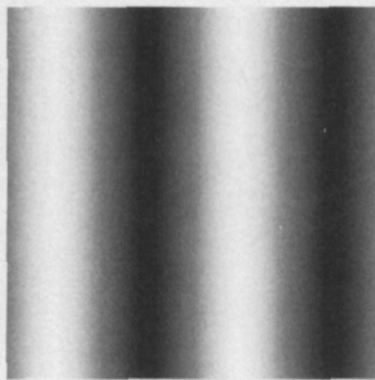
$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

in the rotating
frame of
reference

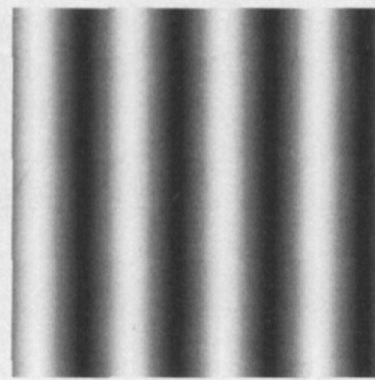
In k -Space
at any given
point of time,
local direction
of M_{xy} is the
phasor in the
above equations.
Spinning at
different rates at
different locations
they accumulate
different phase. 524



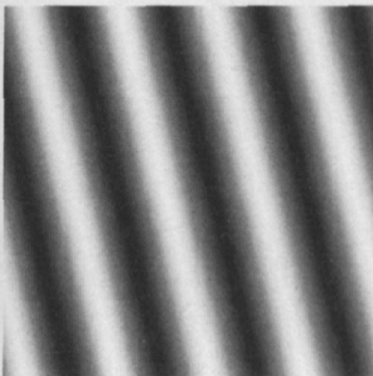
$u_0 = 1, v_0 = 0$



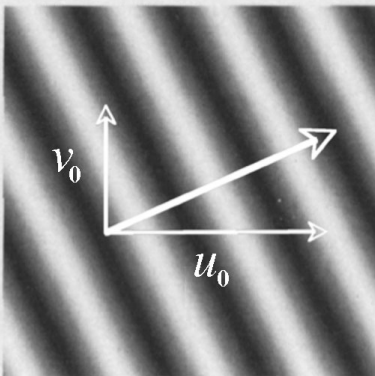
$u_0 = 2, v_0 = 0$



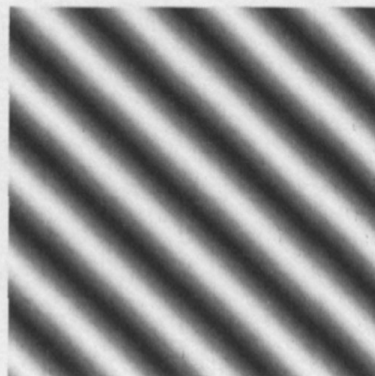
$u_0 = 4, v_0 = 0$



$u_0 = 4, v_0 = 1$



$u_0 = 4, v_0 = 2$

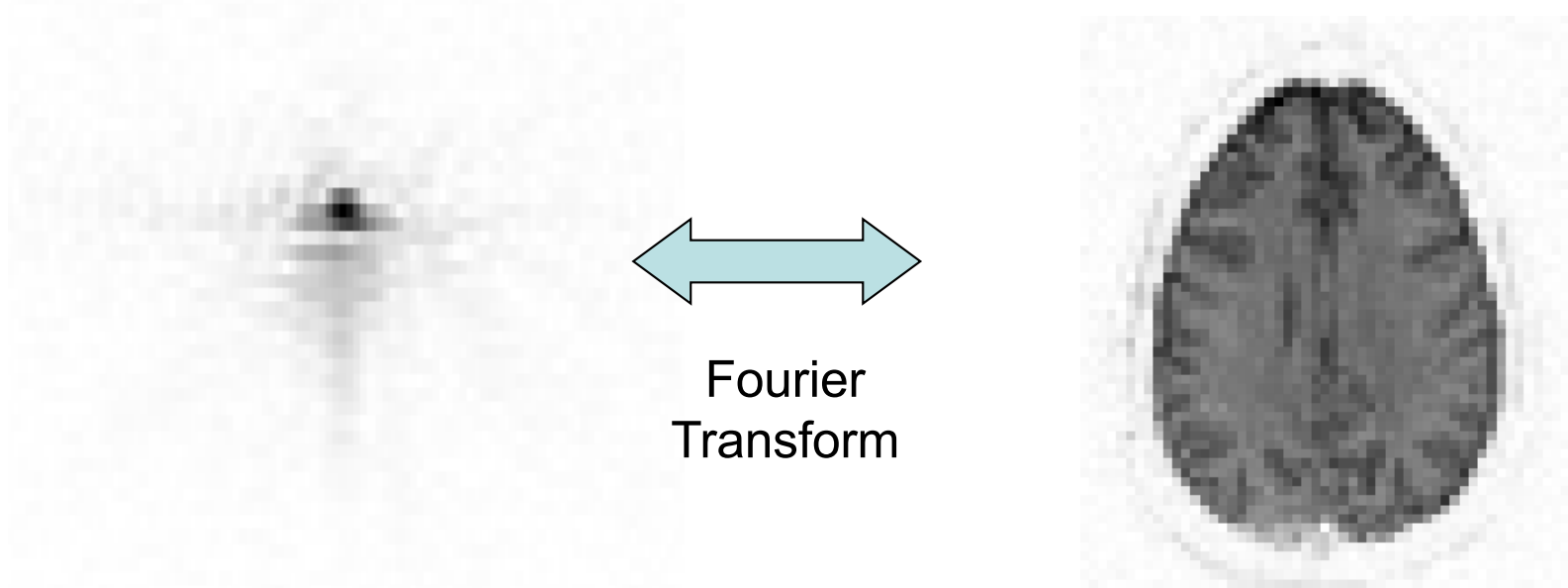


$u_0 = 4, v_0 = 4$

k-Space

k-space

Image Space



-k is the ‘wave number’, the reciprocal of wavelength (number of crests per unit distance).

-Center of k-space represents lowest spatial frequencies (low frequencies contain “bulk” information of the image).

-Frequencies increase as you move from center (high frequencies contain details of image) .

-Spatial frequency in k-space corresponds to the relative phases of transverse magnetization accumulated by applying gradients.

Polar Scanning

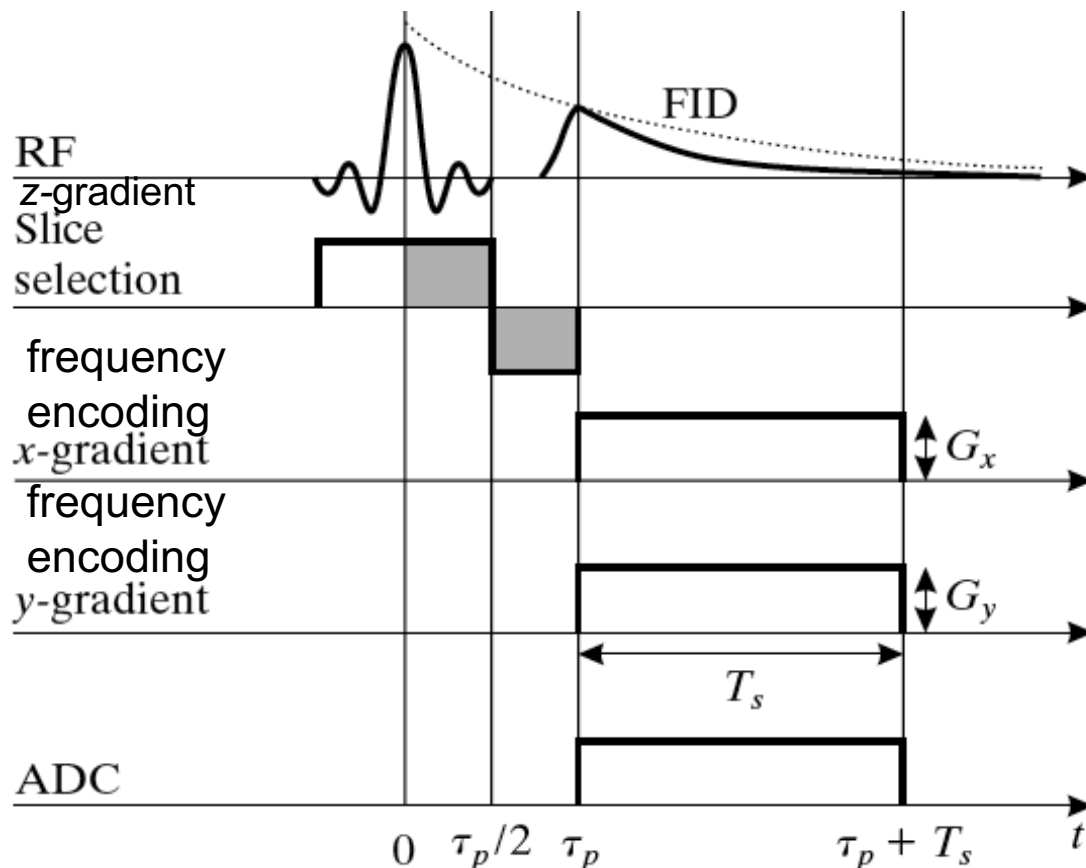
One way is to use x- and y-gradients in the (x,y) plane in a polar pattern

$$v(x, y) = \gamma(B_0 + G_x x + G_y y)$$

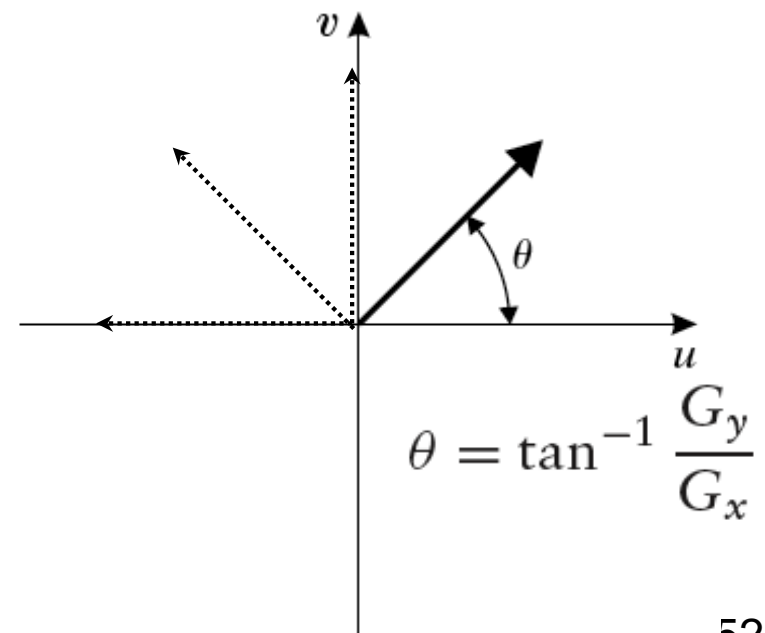
$$s_0(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi\gamma(G_x x + G_y y)t} dx dy$$

$$u = \gamma G_x t$$

$$v = \gamma G_y t$$

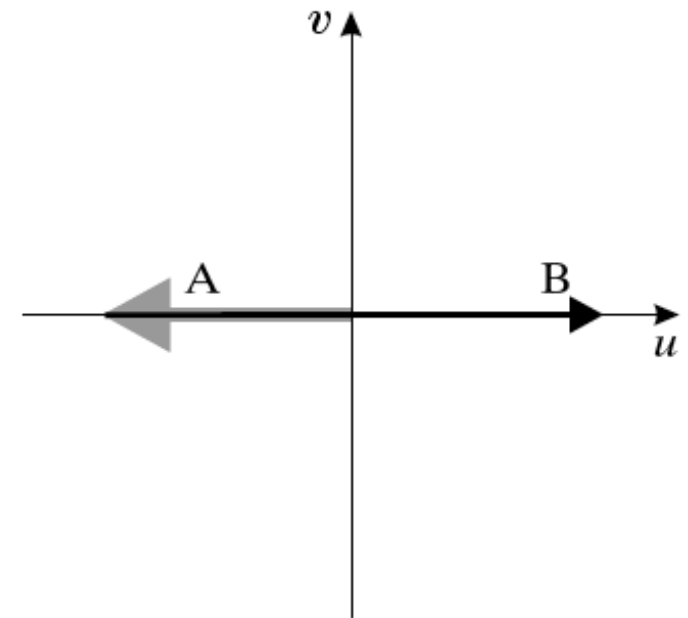
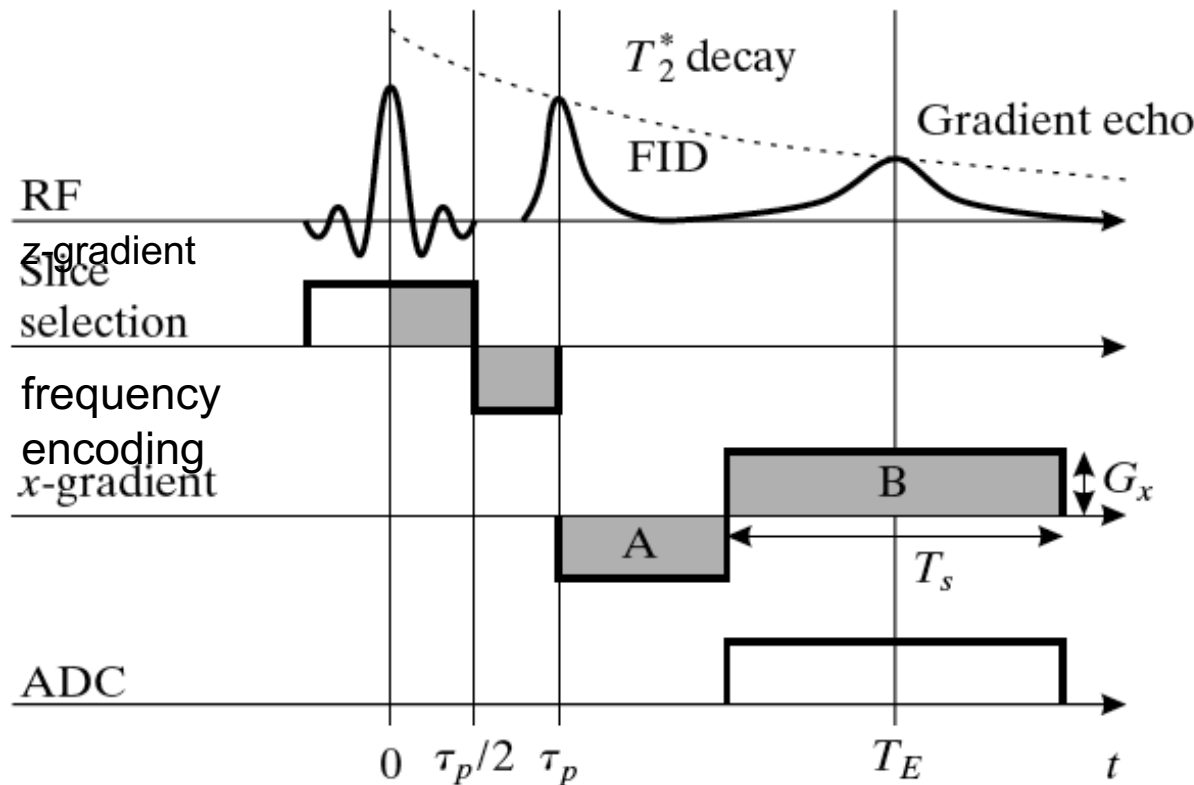


cover all of Fourier space and use filtered backprojection.



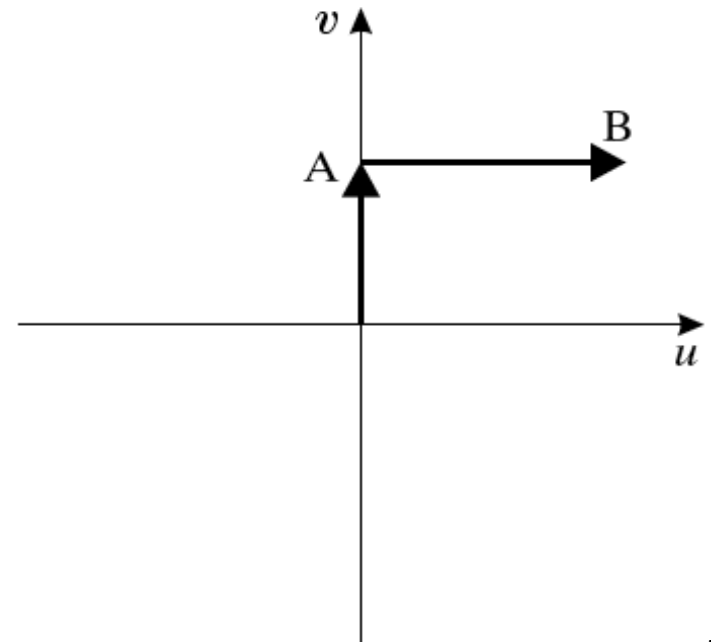
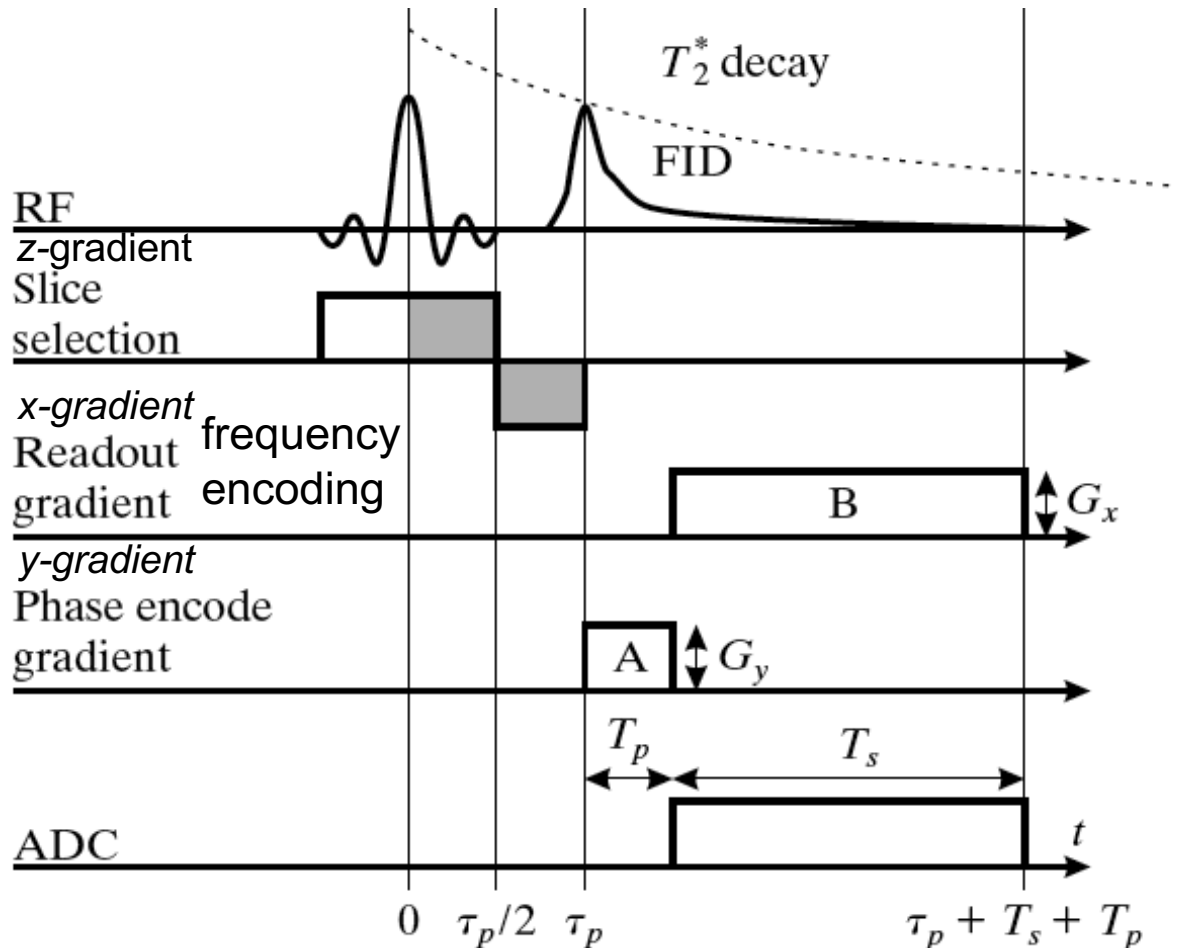
Gradient Echoes

- Negative G_x during period A winds spins backwards as a function of x .
- Positive G_x during period B passes spins through being in phase (echo) and beyond to the same extent in the other direction during acquisition.
- Some spin faster and some slower during acquisition, as a function of x , providing frequency encoding in the x dimension.
- Gradient echoes show transverse relaxation according to T_2^* (as opposed to spin echoes which yield T_2) because spin direction has not reversed, and thus field inhomogeneities are not cancelled.



Phase Encoding

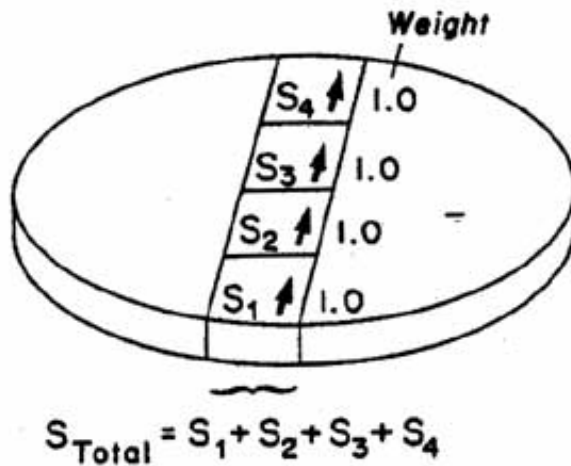
- An extra twist by G_y over interval T_p (p for “phase”) in A (figure below) adds phase as a function of y .
- If this is done repeatedly with different levels of G_y , the entire Fourier space can be covered.



Phase encoding within a frequency band.

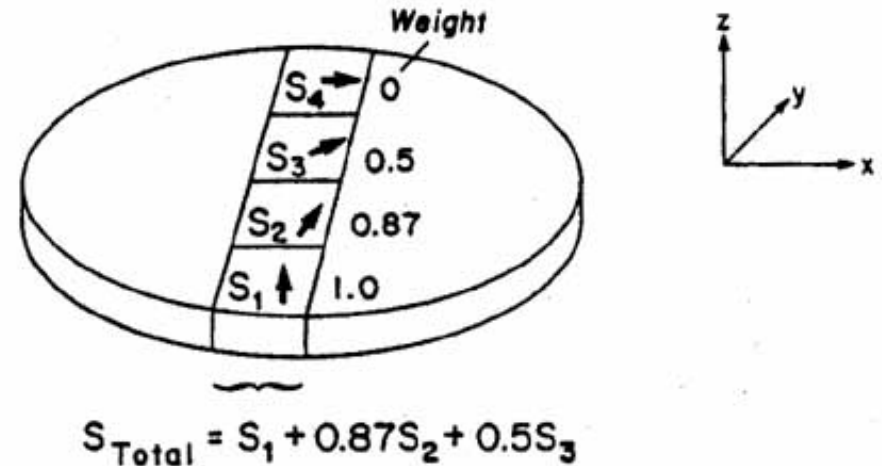
A series each with a different strength for G_y is reconstructed to appear as a frequency proportional to individual twists (S_4 “spins” faster than S_2).

No phase encoding gradient



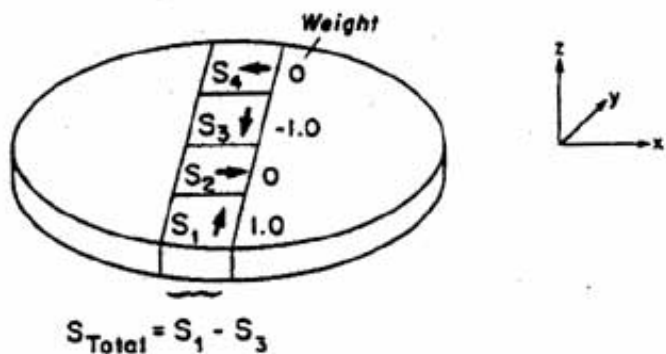
a.

A weak phase encoding gradient

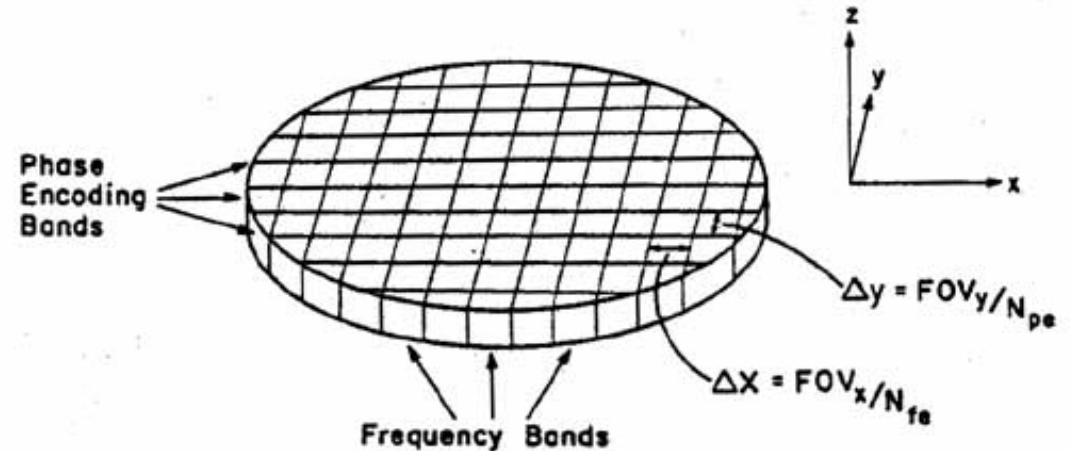


b.

A strong phase encoding gradient

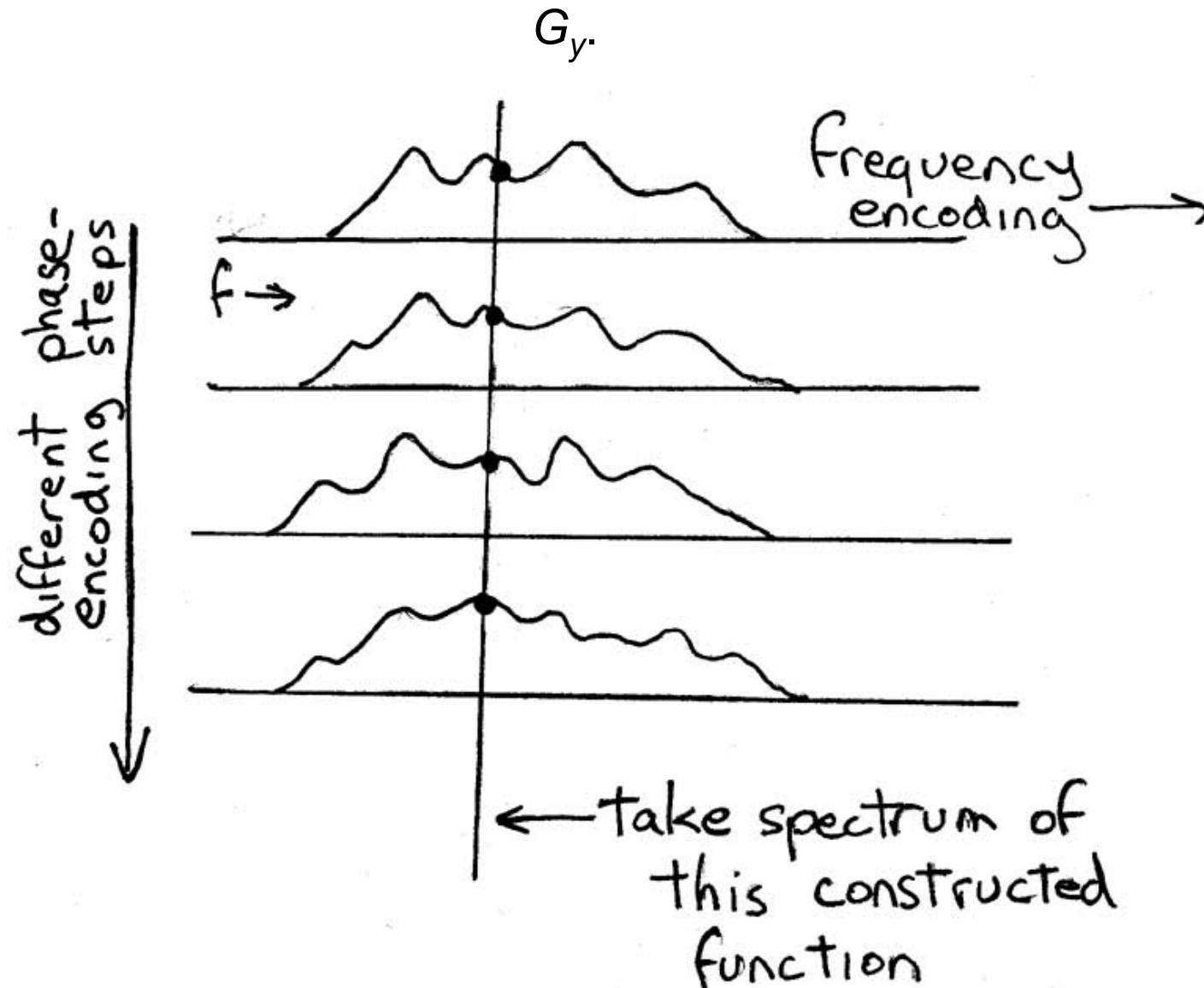


c.



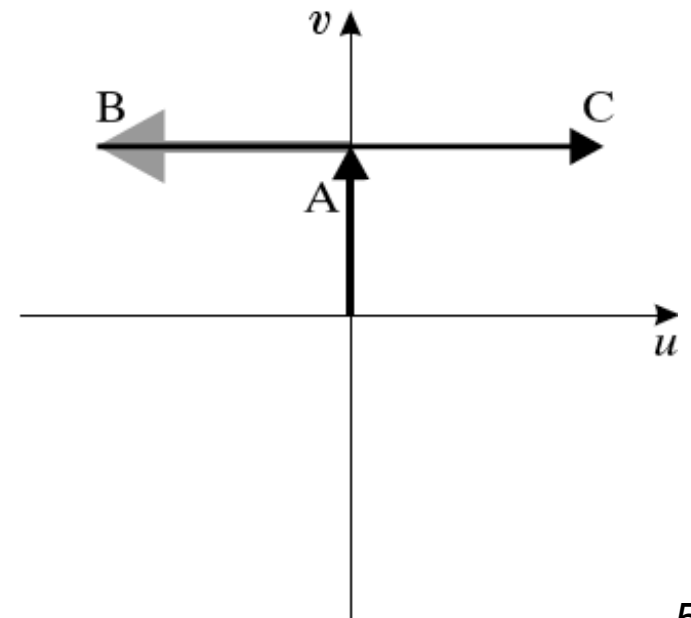
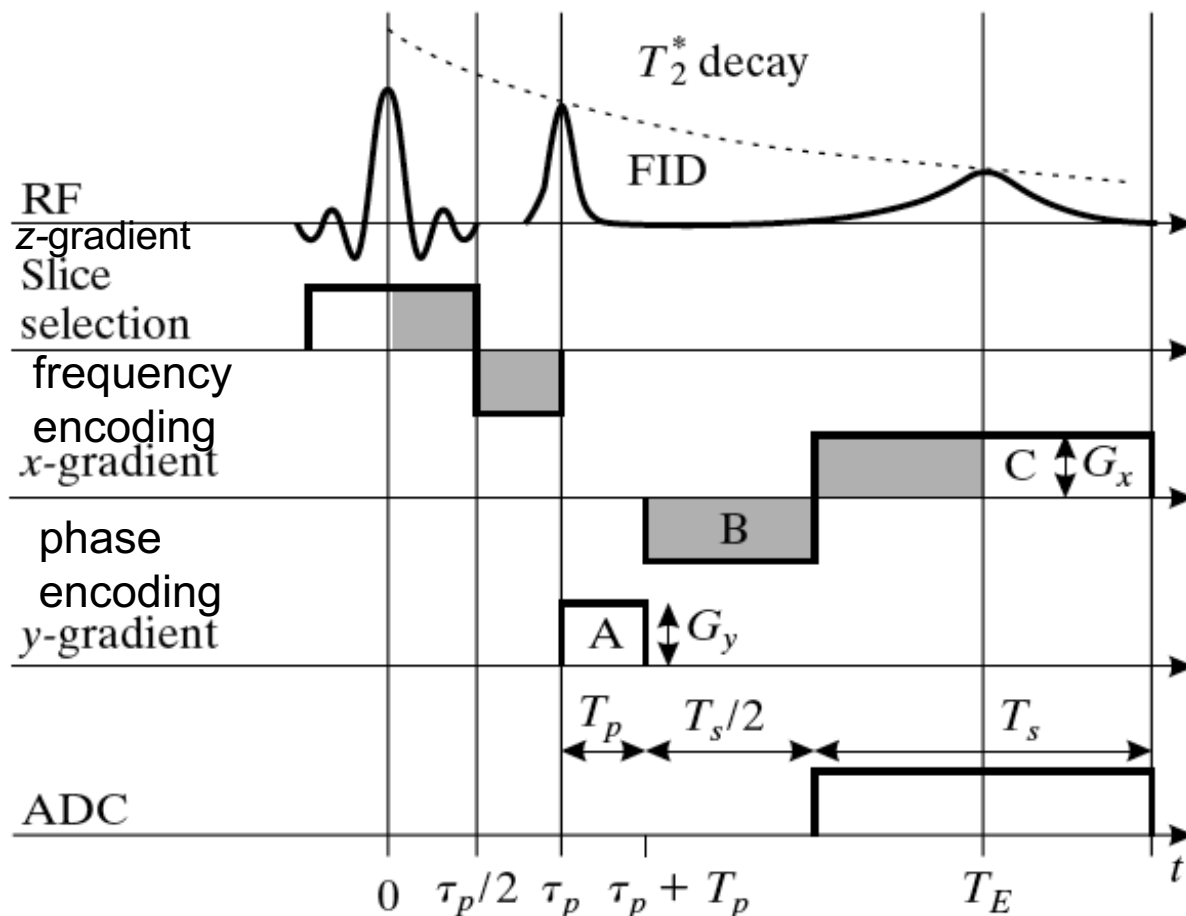
d.

Phase encoding looks at a particular read frequency across multiple acquisitions to construct a function sampled across the different phase-encoded signals. The FT of this “signal” yields “frequencies” each proportional to the phase increment at that point along the phase gradient



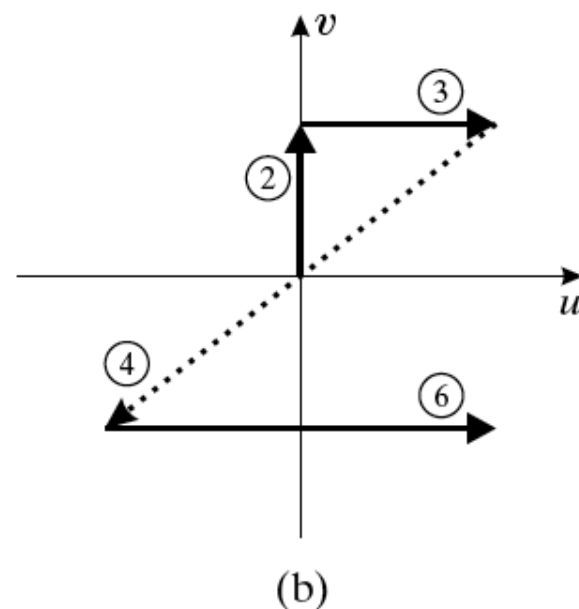
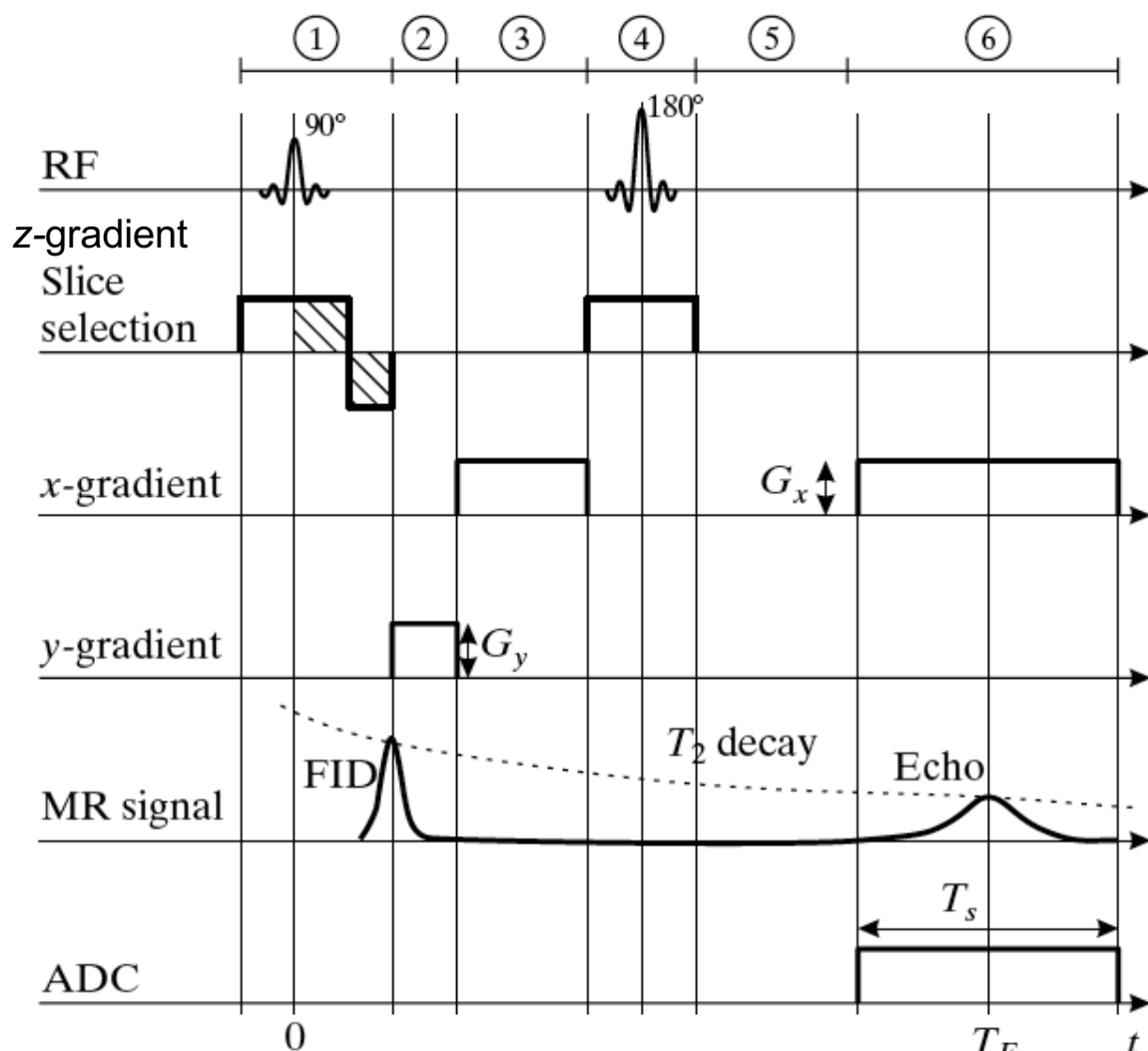
2D Gradient-Echo Pulse Sequence

- The FID in the previous example is not truly captured, except by use of an echo, such as a gradient echo (below) or a spin echo (next slide).
- Here we have a phase-encoded sequence captured with a *gradient echo*.
- As before, gradient echoes yield T_2^* while spin echo (next) yield T_2 .



Spin Echoes

- Here we have a phase-encoded sequence captured with a *spin echo*.
- (1) Slice selection with G_z during 90° RF pulse, refocusing with $-G_z$ (2) Phase encoding gradient G_y (3) wind up for read gradient during spin echo with $+G_x$ positive because of (4) intervening 180° pulse flips spins (5) time to wait for (6) to record spin echo.

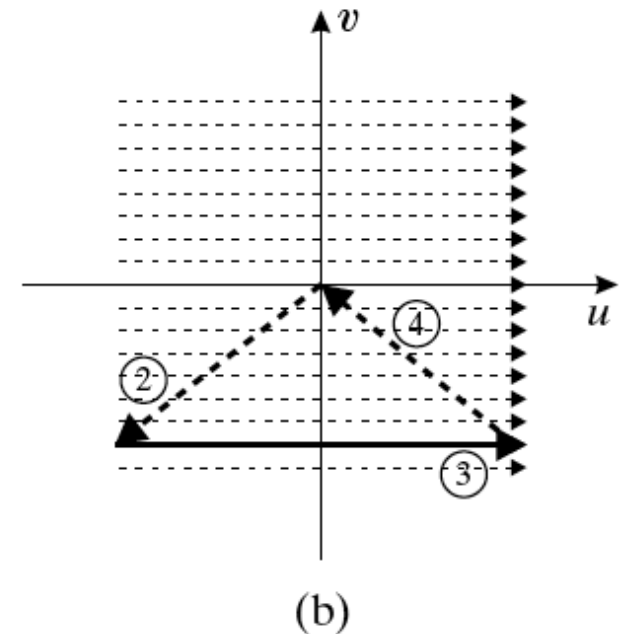
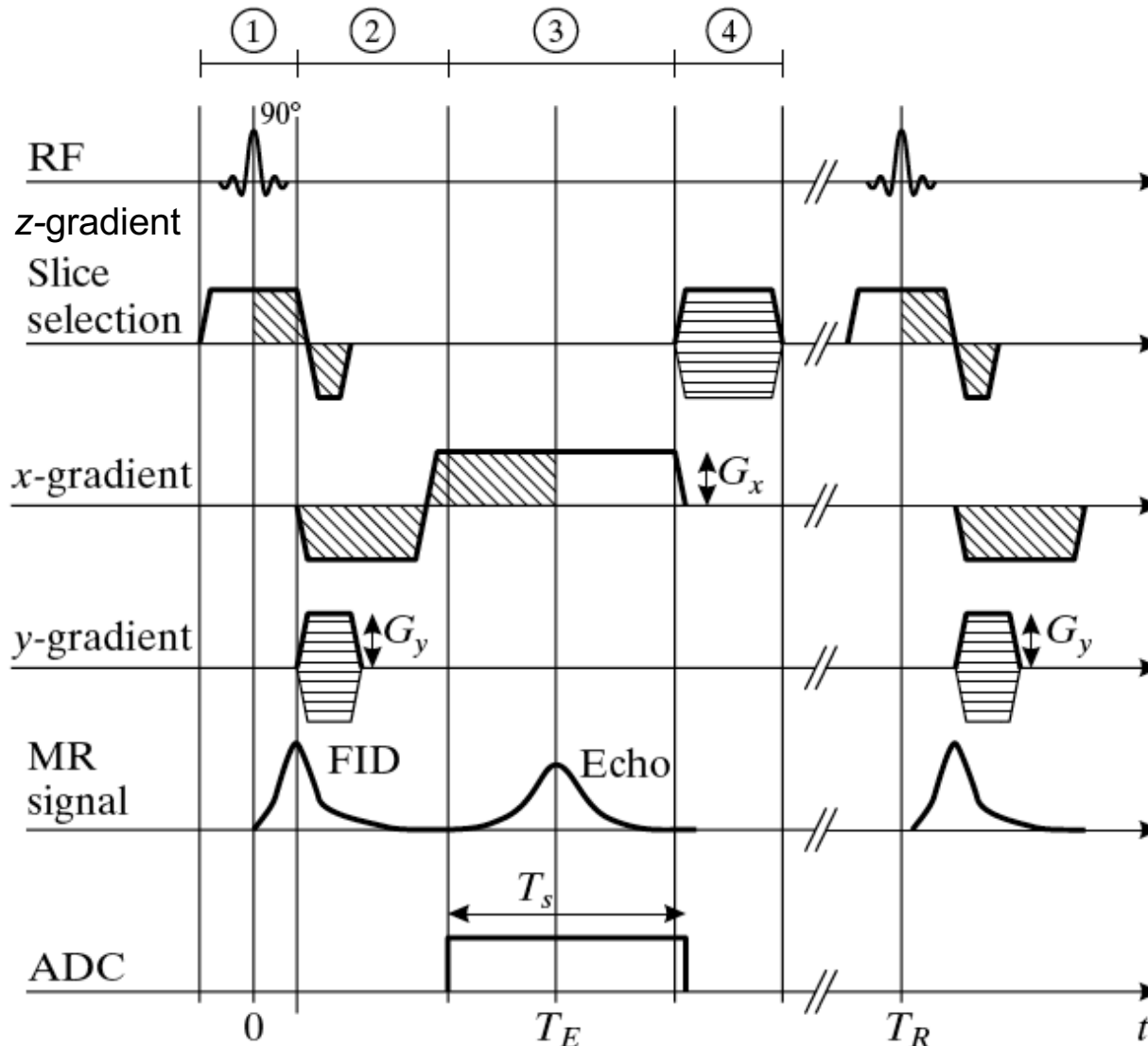


Spoiler pulses to reduce T_R

- In very fast pulse sequences, echoes from previous RF pulses may be confused with echoes from the present RF pulse.
- An extra *spoiler* G_z pulse before each RF pulse makes spins within a slice go out of phase, eliminating previous signals.
- The strength of the spoiler pulse is randomized to prevent the phases from getting back in phase due to chance combinations.
- Gradient echo sequences using this technique are called *spoiled gradient echo* (SPGR) image sequences.

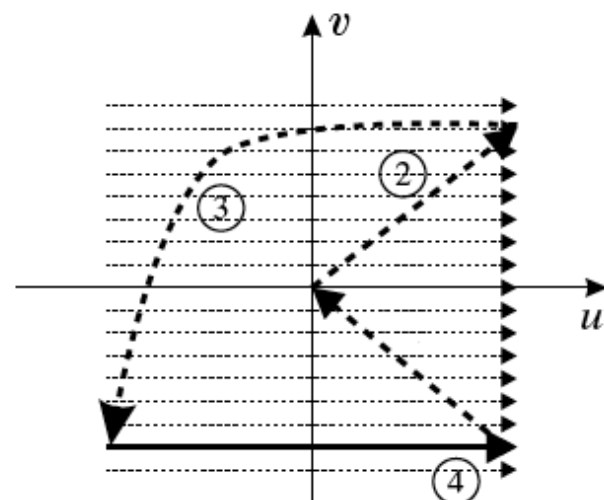
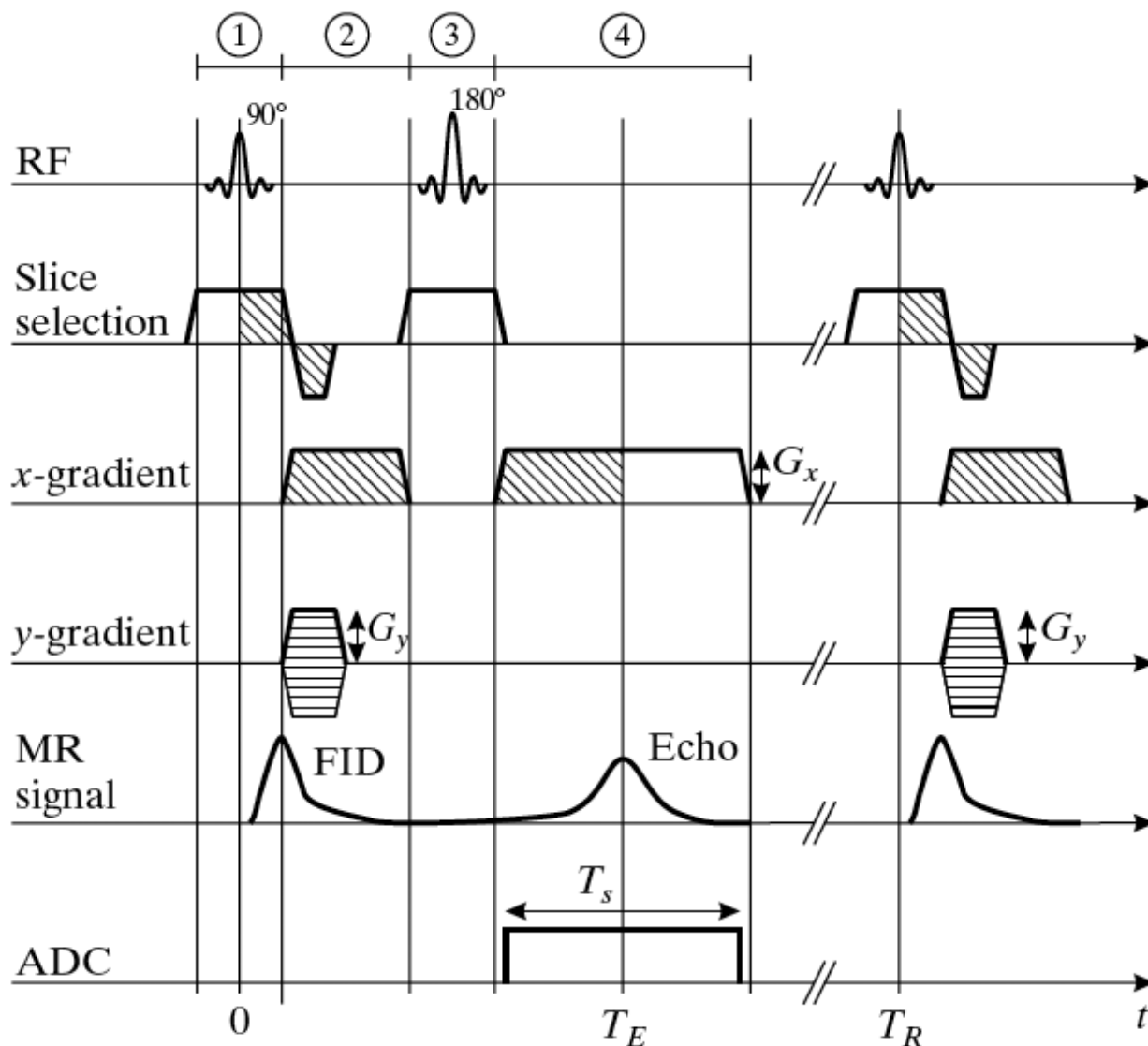
Realistic 2D Gradient Echo Pulse Sequence

- Notice all gradient pulses are trapezoidal at realistic slew rate; (1) 90° RF pulse with G_z slice selection (2) simultaneous non-interfering refocusing with G_z , windup for gradient echo with G_x , and phase encoding with G_y (3) reading signal during gradient echo (4) randomized G_z spoiler pulses.



Realistic 2D Spin Echo Pulse Sequence

- (1) 90° RF pulse with G_z slice selection (2) simultaneous refocusing with G_z , windup for read gradient during spin echo echo with G_x , and phase encoding with G_y (3) 180° RF pulse to flip spins with G_z slice selection (4) reading the spin echo with frequency encoding by G_x



(b)
Note corrected
numbering
from Fig. 13.19
in book

2D Polar Spin Echo Pulse Sequence

- Permutations of x - and y -gradient strengths to effect different angles (2) during windup for frequency encoding during spin echo and (4) during acquisition of signal during spin echo.

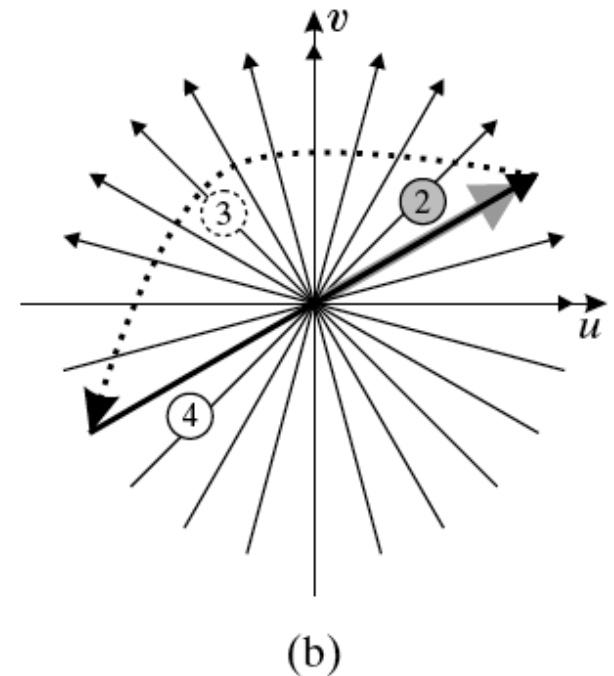
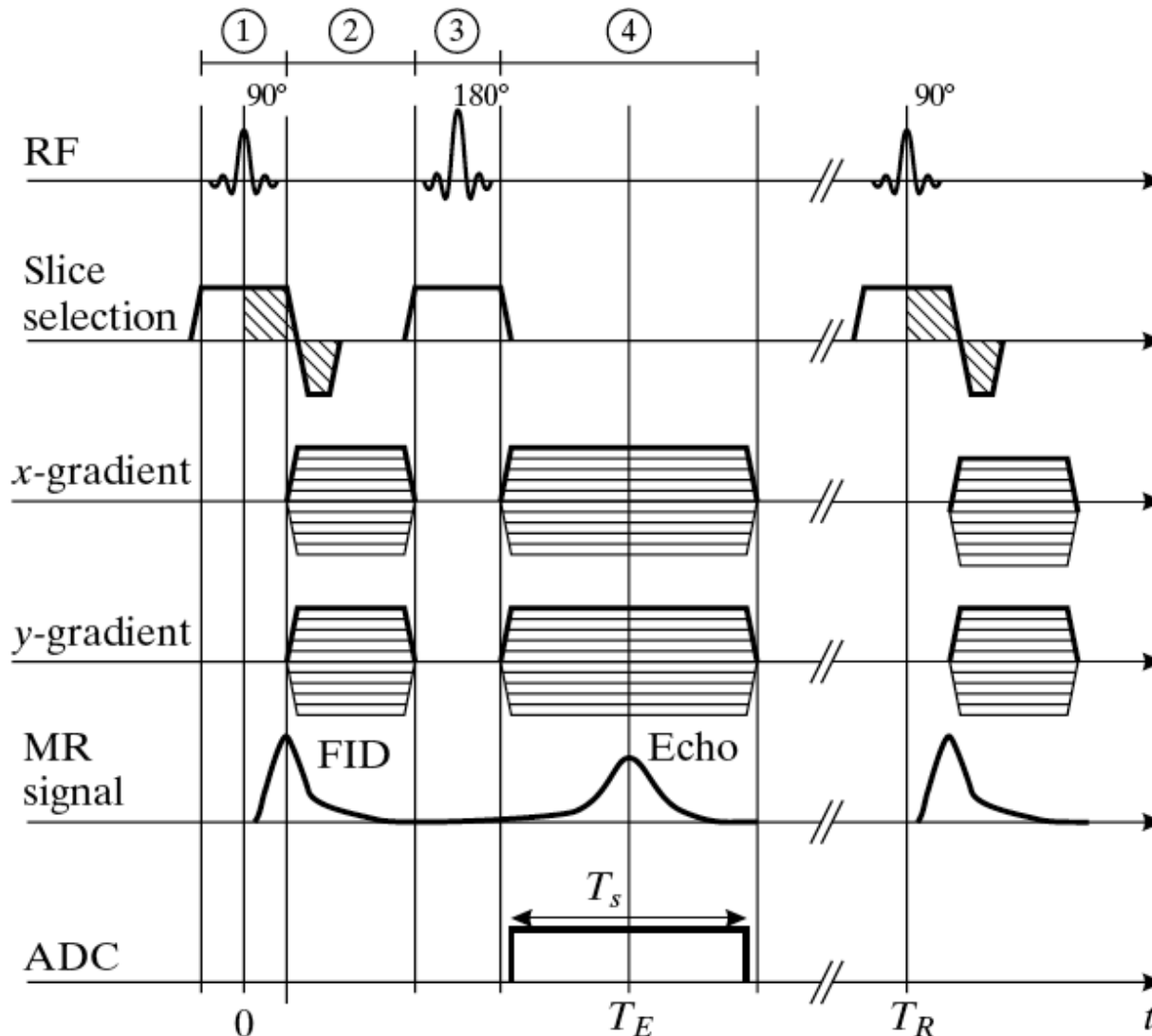


Image Reconstruction and Resolution

- With *rectilinear* traversal of k -space (phase encoding with either gradient-echo or spin-echo), a simple inverse Fast Fourier Transform (FFT) works.

$$u = \gamma G_x t,$$

$$v = \gamma A_y,$$

area of phase encode gradient $G_y T_p$

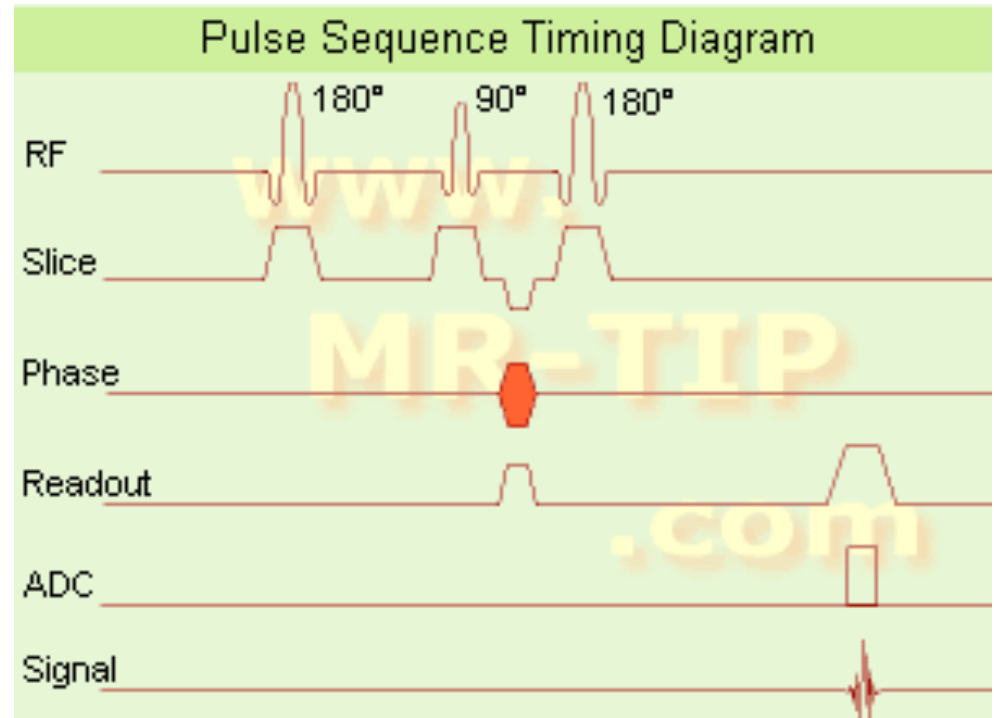
- Analog filtering in the u (frequency encoded) direction prevents aliasing
 - No such filtering possible in the v (phase encoded) direction, so image can “wrap around” if objects exist outside the field of view.
- With *polar* traversal of k -space we use *filtered backprojection* or *convolution backprojection*, as we did in CT.
- In both, resolution is limited by the extent of frequencies represented in k -space.

Other Image Characteristics

- Signal to Noise
 - Increased by: Larger proton density P_D , larger static magnetic fields B_0 , lower temperature, surface RF coils, larger pixel volume, $\pi / 2$ RF pulse, greater acquisition time for signal.
- Artifacts (besides wrap-around in phase direction)
 - *Geometric Warping* due to non-uniformity in the gradient strength.
 - *Ghosting* due to motion from breathing, swallowing, tremor, heart beat, between Fourier space acquisitions. Reduced by breath-holding and gating to the ECG.
 - *Chemical Shift* due to differences between the Larmor frequencies of fat and water, making them show up at slightly different locations along the frequency encoding gradient.
 - *Intensity Variation* due to RF field inhomogeneity.

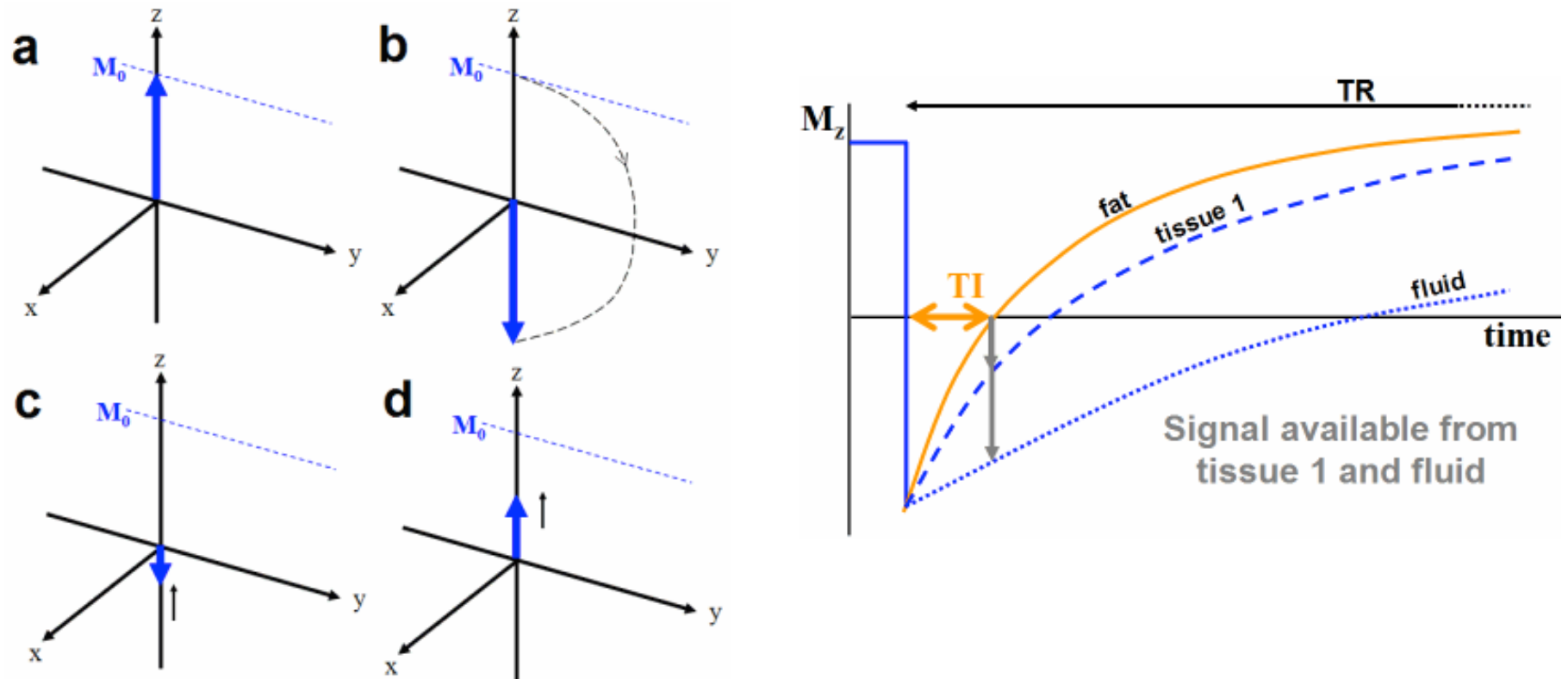
Inversion Recovery

- Starts with a 180° RF inversion wave which flips longitudinal magnetization M_z in the opposite direction (negative).
- Due to longitudinal relaxation, longitudinal magnetization will return to its initial value, passing through null value.
- To measure the signal, a 90° RF wave is applied to obtain transverse magnetization (positive, null, or negative)
- The delay between the 180° RF inversion wave and the 90° RF excitation wave is referred to as the inversion time TI or “tau” (inversion time).



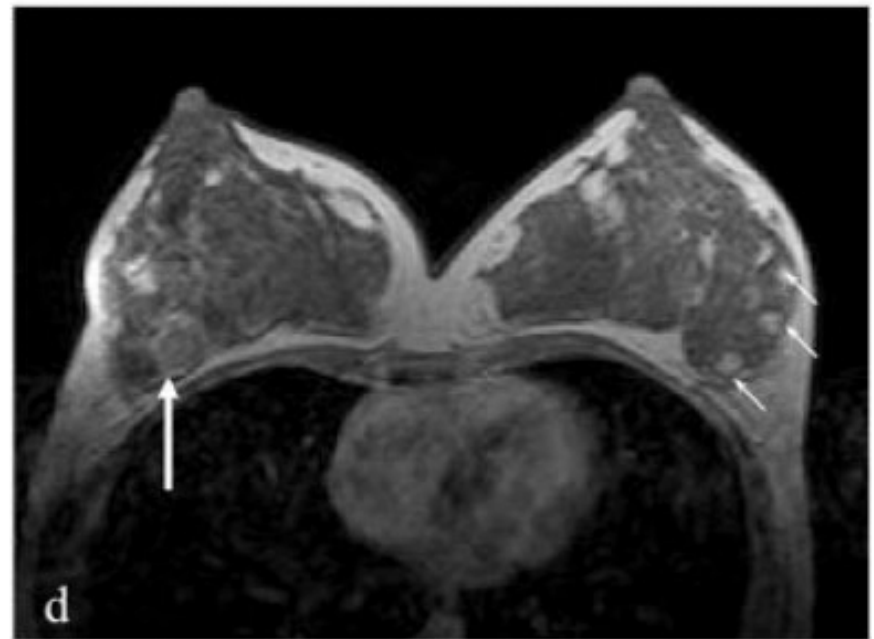
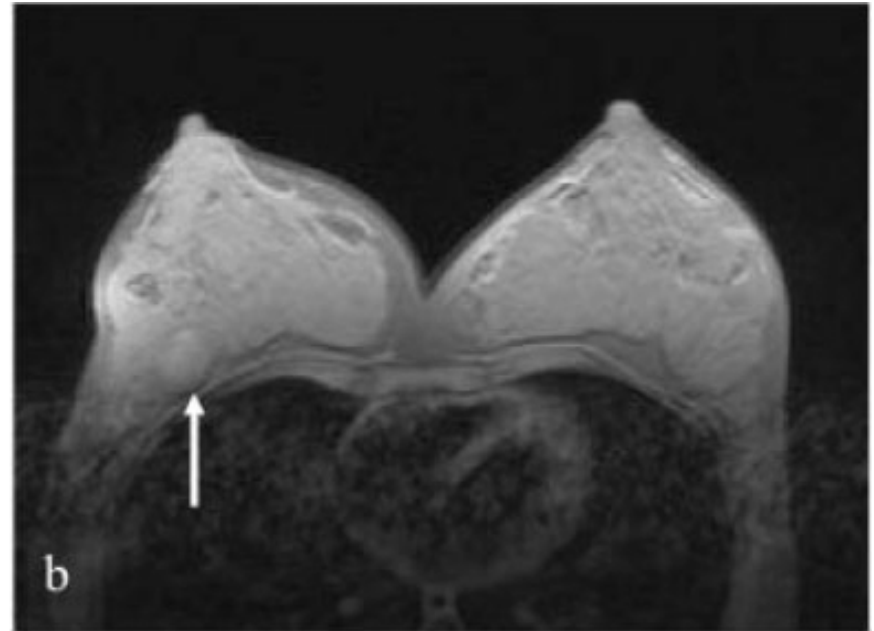
Short Tau Inversion Recovery (STIR)

- TI (tau, or the inversion time) adjusted so that fat is null.

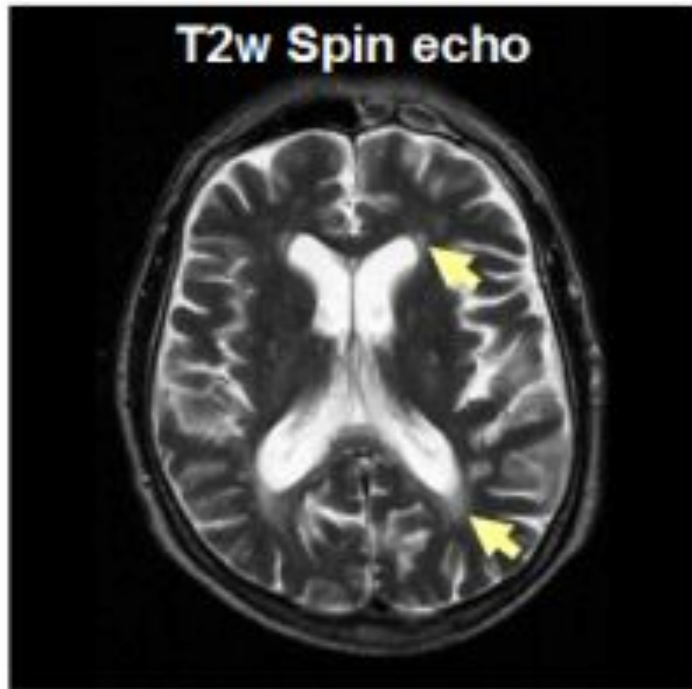


STIR (continued)

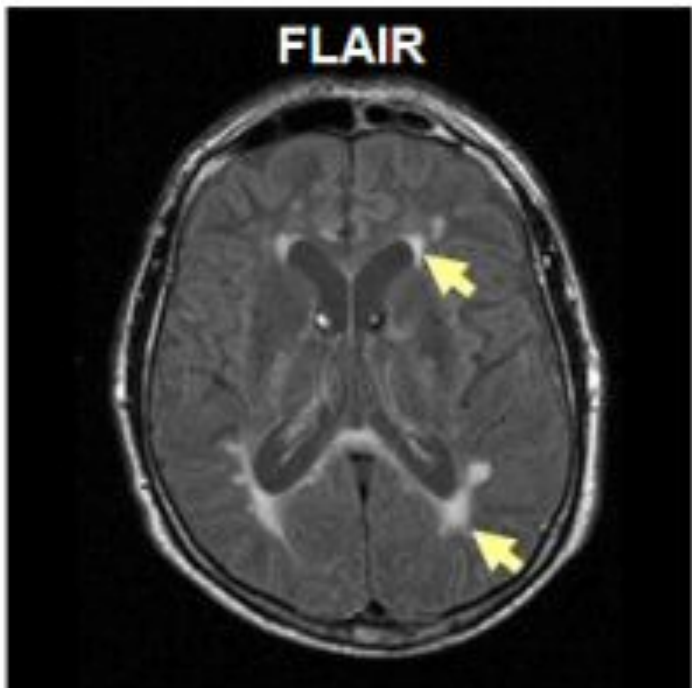
- (b) MRI study of the breasts showing medullary carcinoma in the right breast (long white arrow) appearing isointense to fat on T1 weighted image.
- (d) With STIR nullifying fat, carcinoma is clearly visible along with three enhancing fibroadenomas in the left breast (short white arrows).



Fluid Attenuation Inversion Recovery



- The TI (inversion time) of the FLAIR pulse sequence is adjusted to the relaxation time of water, nullifying fluid.
- Lesions that are normally covered by bright fluid signals using conventional T2 contrast are made visible.



<< FLAIR image, revealing periventricular lesions more clearly because the adjacent fluid signal is suppressed.

Gadolinium

- Element that strongly decreases T1 of nearby protons.
- Rare but severe side effects: allergic reaction, kidney failure...
- Doesn't normally cross blood brain barrier, except in pathology

Multiple Sclerosis

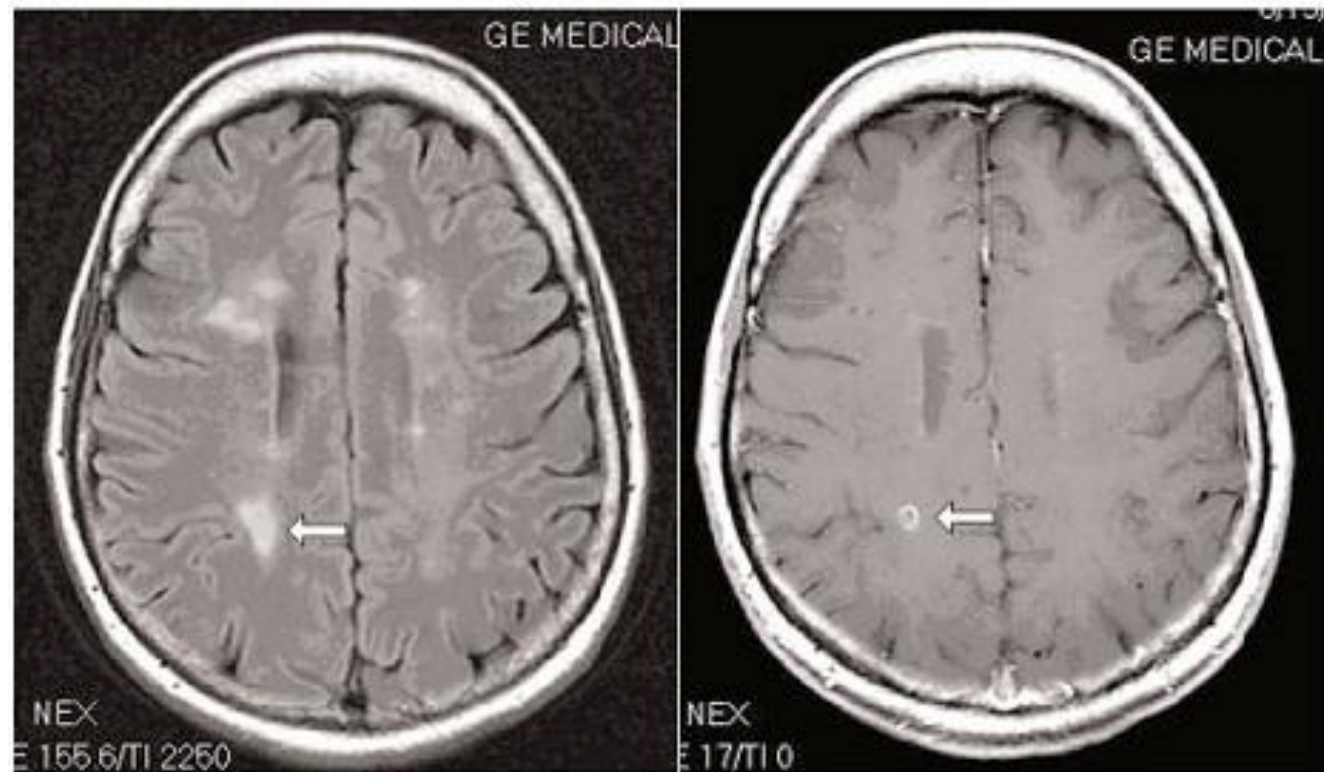


Figure 1 The left panel shows a FLAIR image with several typical T2 lesions (scars). The right panel shows the same level of the brain with a T1 weighted image after Gadolinium enhancement. The arrow demonstrates which one of the lesions was active at the time of the MRI.

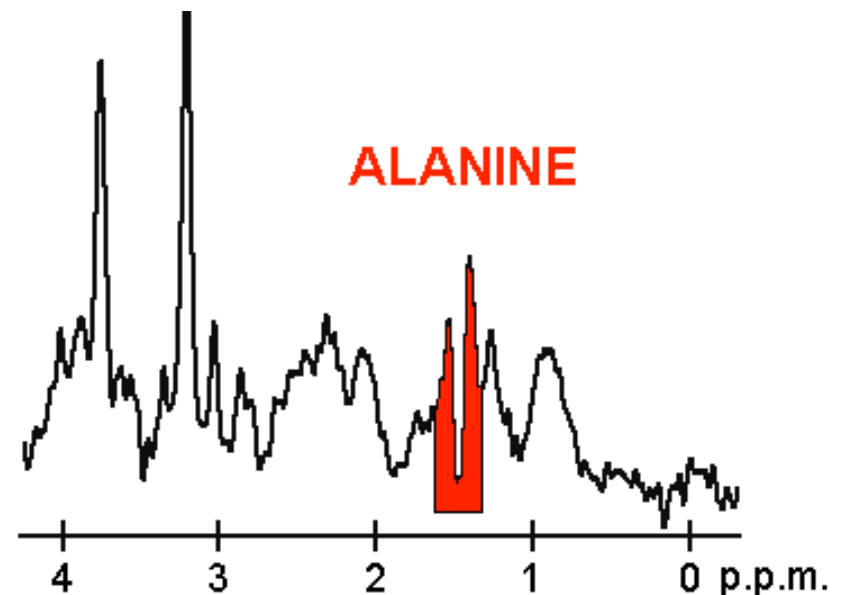
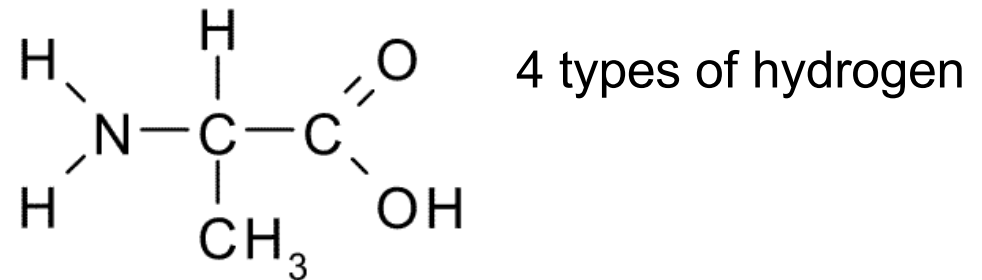
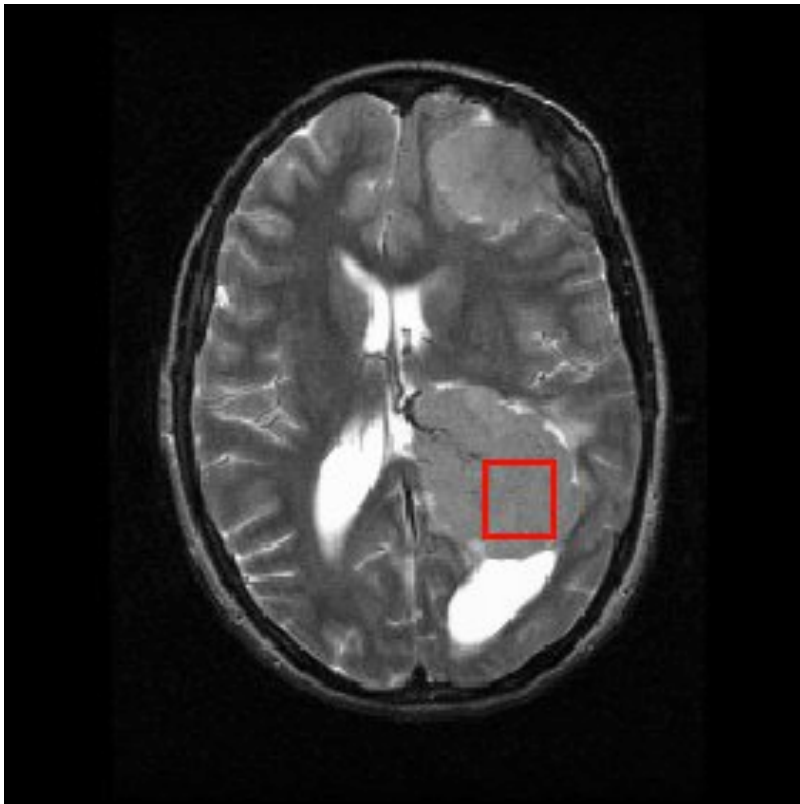
MRA (Magnetic Resonance Angiography)

- A number of different methods
 - Contrast Enhanced (CE) MRA. Contrast agent (gadolinium) makes vessels stand out.
 - Time of Flight (TOF) or Inflow MRA. Blood in slice is saturated (by repeated pulses with TR much shorter than T1), so that only fresh blood entering slice gives off signal. No contrast agent. Repeating in all 3 directions can yield 3D flow.



MRS (Magnetic Resonance Spectroscopy)

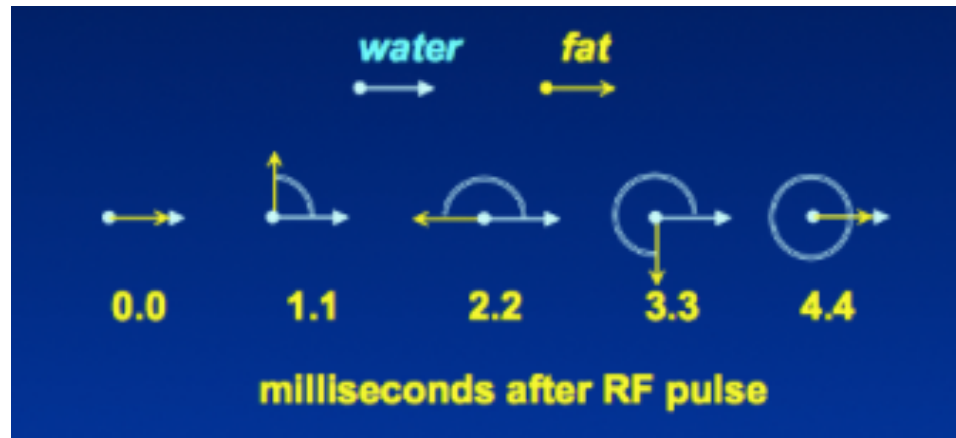
- Displays the proton spectrum for a region of the MRI scan.
- Each biochemical has a different peak at a known frequency.
- Spatial resolution of MRS is much lower than MRI.
- Depends on chemical shift “artifact.”



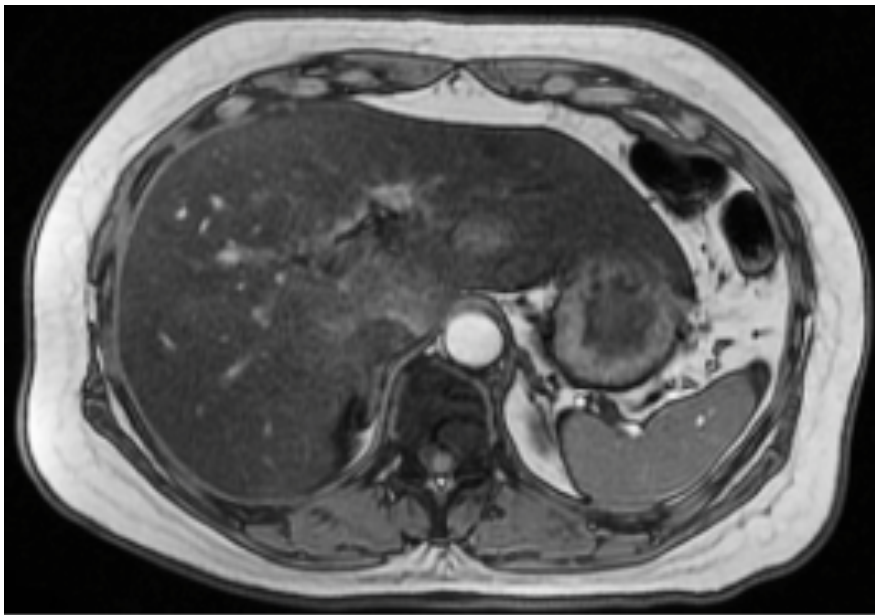
In-Phase/Out-of-Phase Images (fat/water)

- Water and fat *in a given voxel* have slightly different Larmor frequencies, due to chemical shift.
- Over time, they go in and out of phase, relative to each other.
- Multiple gradient echoes can be elicited to differentiate in-phase (bright) from out-of-phase (darker).
- Can divulge high fat content, such as in an abnormally fatty liver.

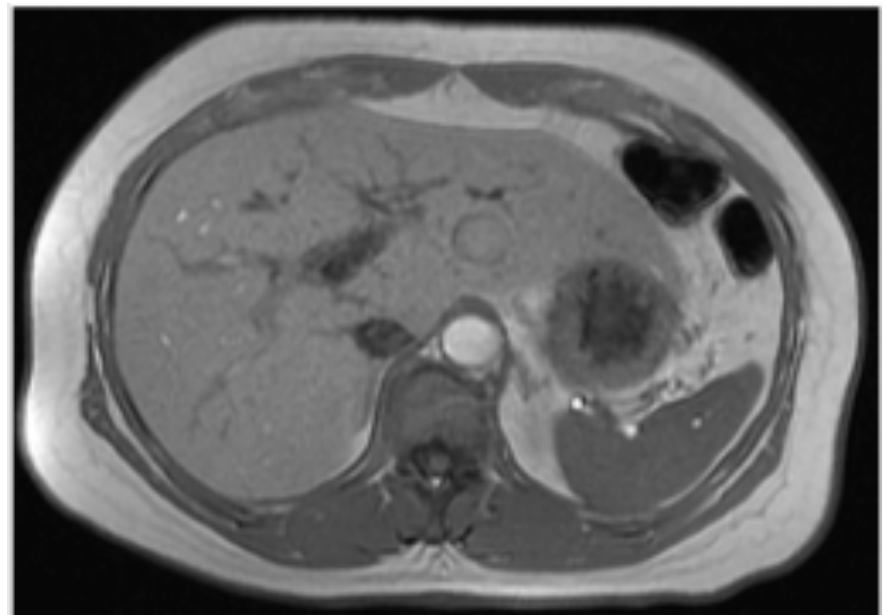
In-Phase/Out-of-Phase Images of Fatty Liver



Phase cycling of fat relative to water at 1.5 T



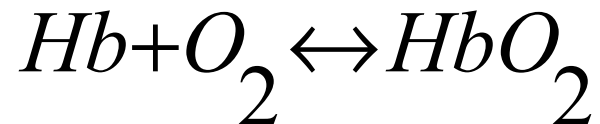
Out-of-phase (2.2 ms)



In-phase (4.4 ms)

BOLD (Blood oxygen level-dependent) MRI

- T2* decreases with greater *magnetic susceptibility* due to greater B₀ inhomogeneity.
- Neuronal activity leads to a local increase in blood flow and therefore increased HbO₂ (oxygenated hemoglobin).

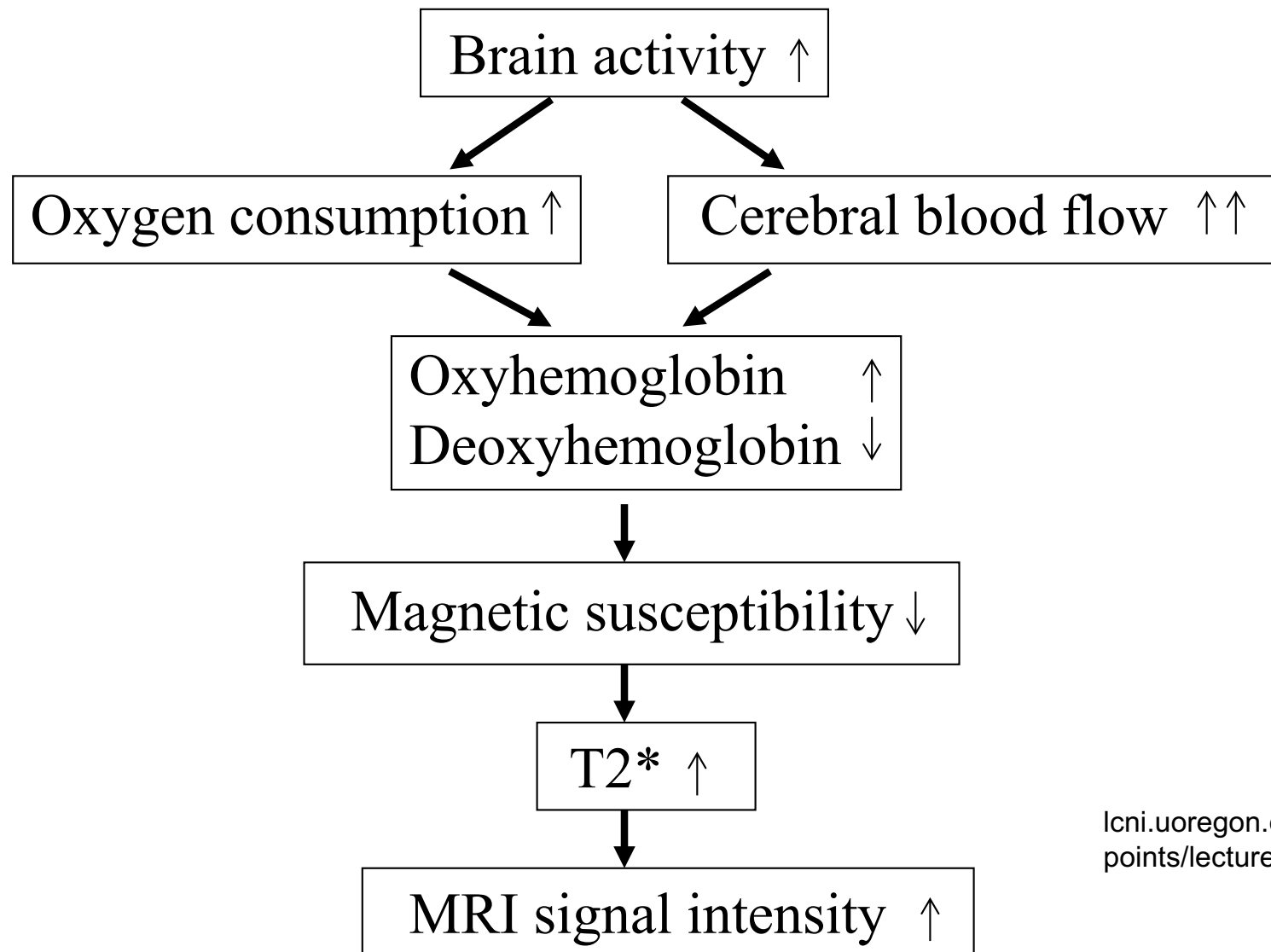


Deoxyhemoglobin: paramagnetic with respect to the surrounding tissue (increased magnetic field) leads to increased magnetic susceptibility and shorter T2*.

Oxyhemoglobin: diamagnetic with respect to the surrounding tissue (**decreased same** magnetic field) leads to decreased magnetic susceptibility and longer T2*.

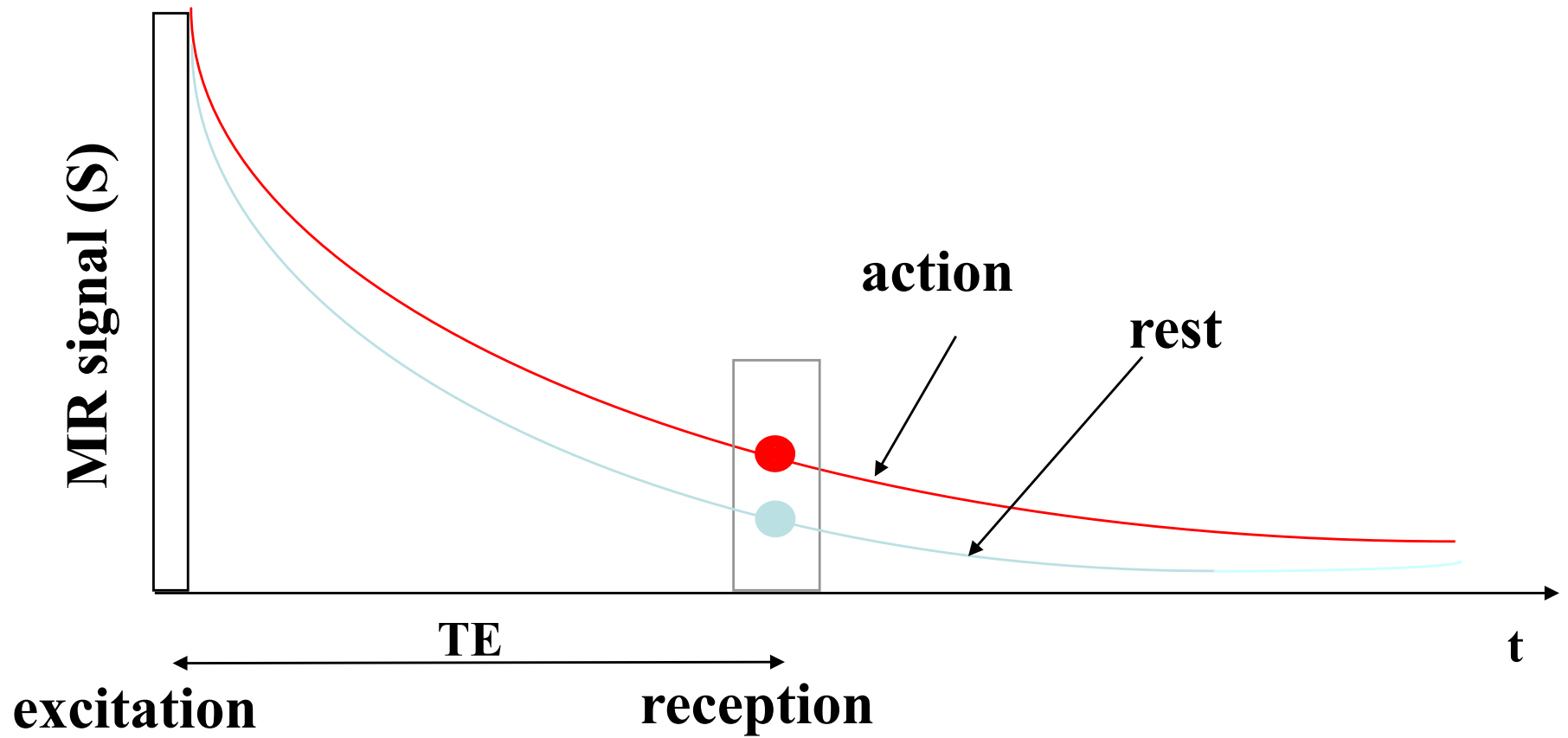
Diamagnetism is weaker than paramagnetism and is present in all materials.

Mechanism of BOLD fMRI



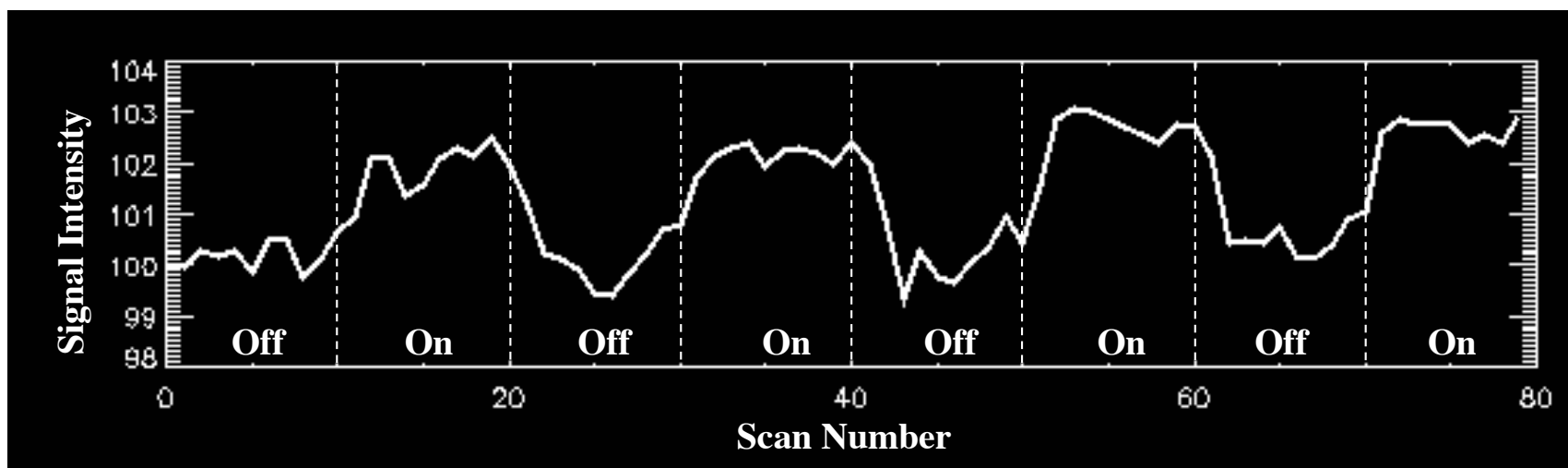
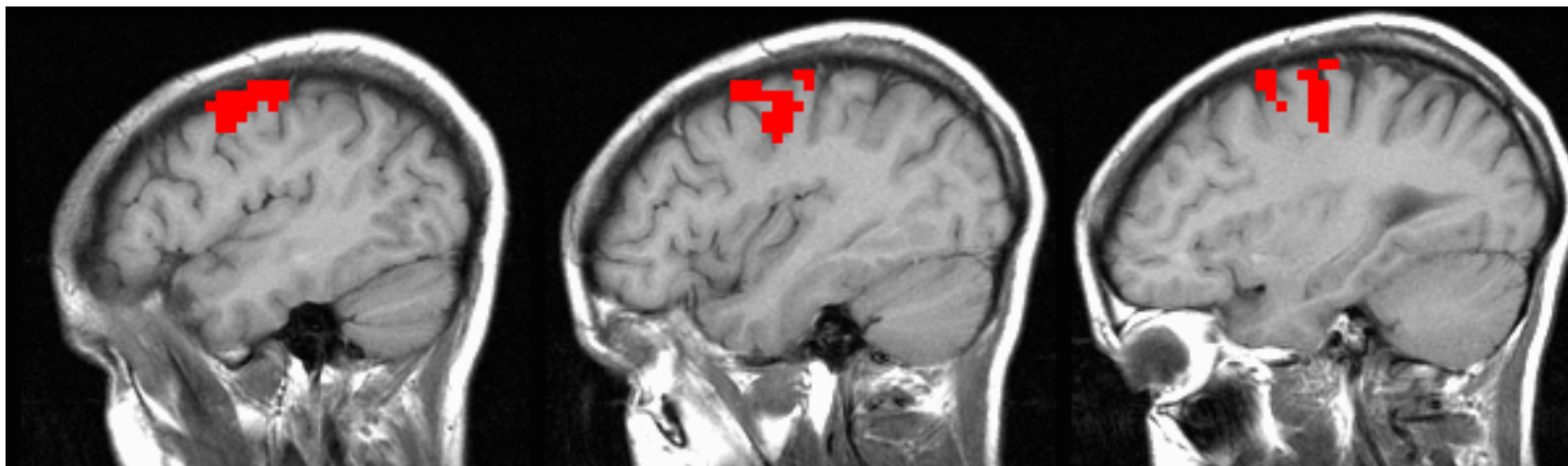
lcn.uoregon.edu/~ray/power
points/lecture_10_24.ppt

T_2^* Effect in fMRI



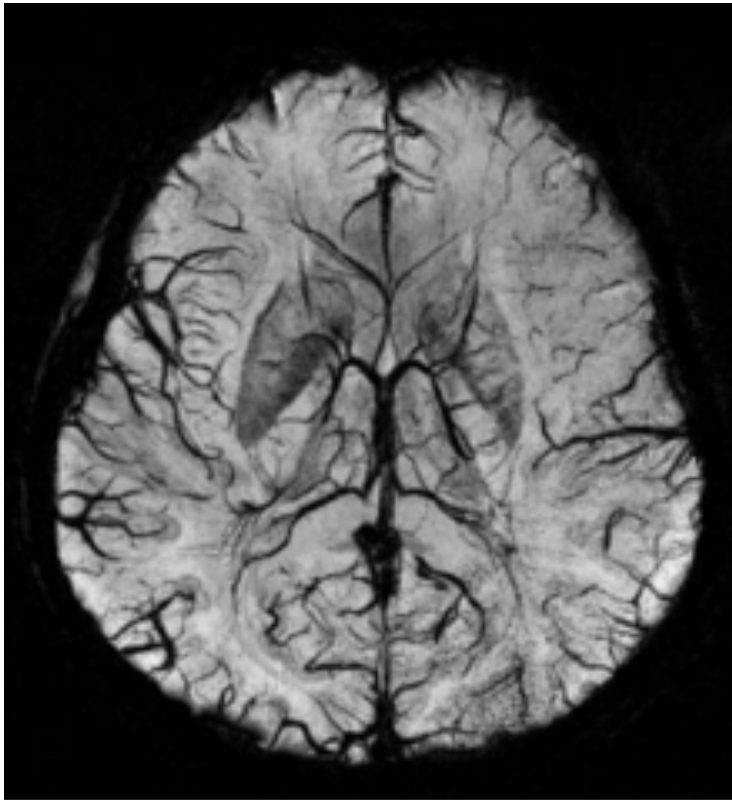
BOLD fMRI

Time Series and Activation Maps

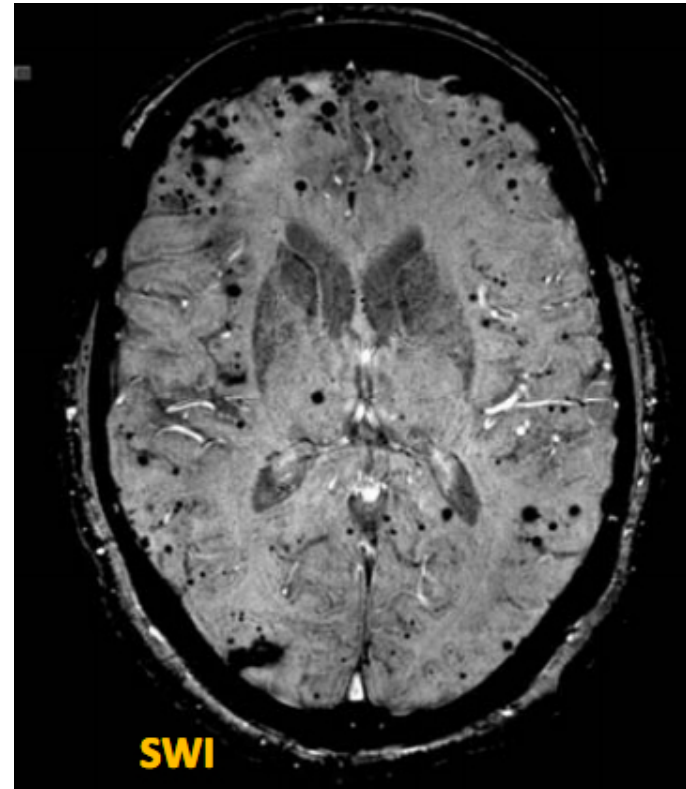


Susceptibility weighted imaging (SWI)

- “BOLD venography”
- Very sensitive to venous blood, hemorrhage and hemosiderin (iron from old bleeds). Their diamagnetic properties increase B_0 .
- Long echo, gradient recalled echo (GRE) intentionally sensitive to $T2^*$.
- Phase differences used to see changes in Larmor frequency due to susceptibility.



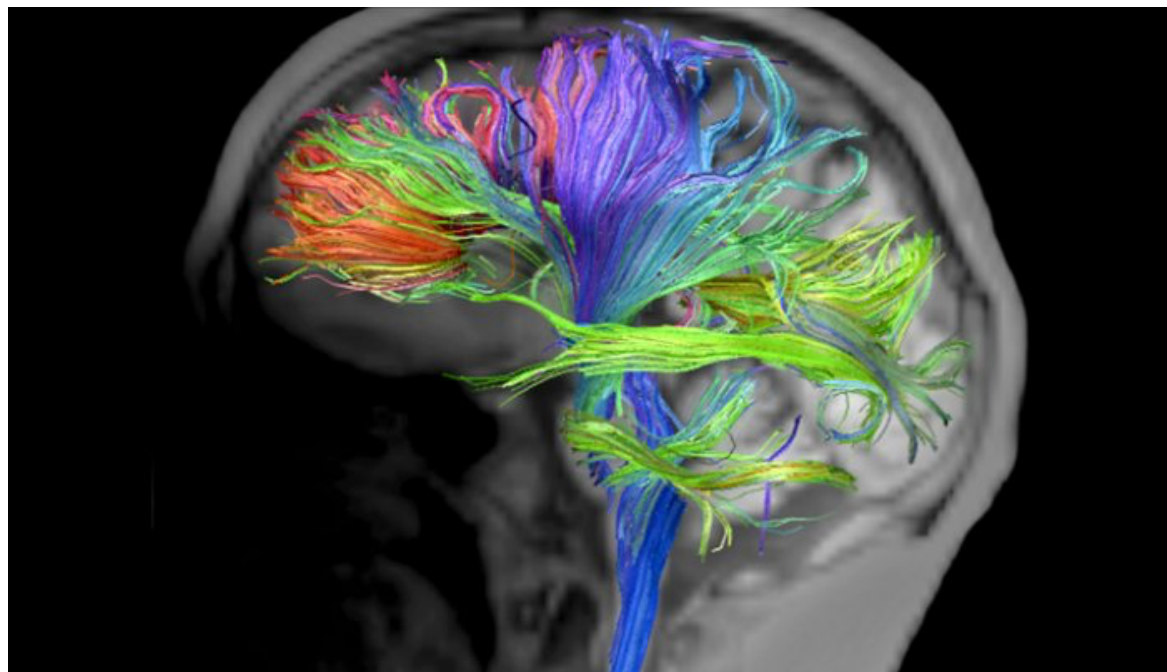
veins



hemosiderin

DTI (Diffusion Tensor Imaging)

- Water diffuses more quickly along axons because myelin is waterproof.
- A pair of strong gradient pulses are added to the pulse sequence.
- The first pulse de-phases the spins, and the second pulse re-phases the spins if no net movement occurs.
- If net movement occurs between the gradient pulses, signal attenuation occurs because the spins don't completely rephase.



Applying diffusion gradients in at least 6 directions, it is possible to calculate a tensor (i.e. a 3 x 3 matrix) that describes the 3-dimensional anisotropic shape of diffusion. DTI allows us to visualize the location, orientation and anisotropy of the brain's white matter tracts. Permits analysis of the "connectome."