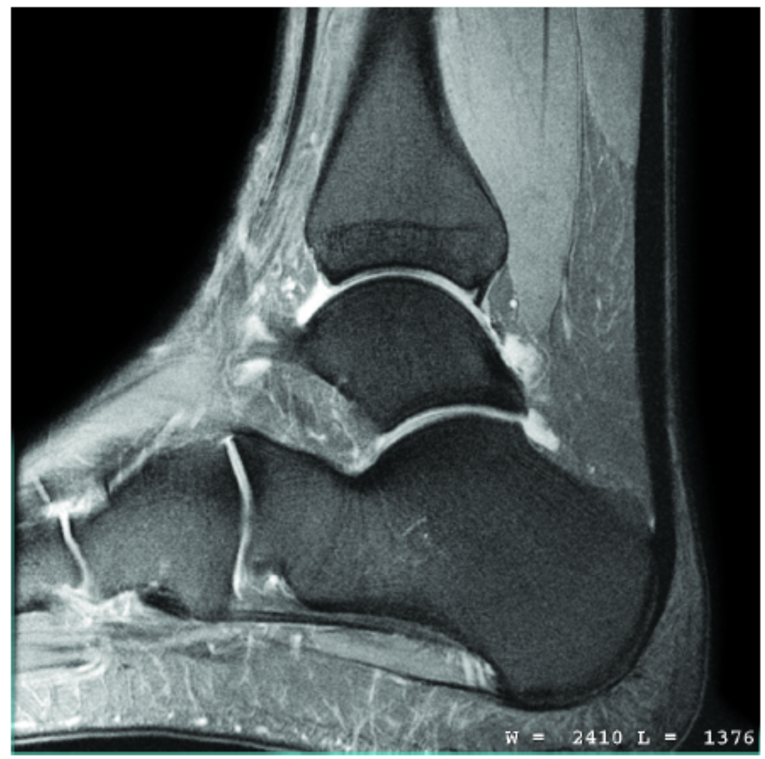
Chapter 12
Physics of Magnetic Resonance

- Magnetic Resonance Imaging
  - High resolution
  - High contrast
  - Tomographic
  - Non-invasive (non-ionizing)
- Nuclear Magnetic Resonance (NMR) properties
  - Stimulated by magnetic fields and radio-frequency fields
  - *Pulse sequences* govern time varying application of these fields
- Mainly for anatomy but can see function as well
  - Blood flow
  - Diffusion of water
  - Blood oxygenation: *functional MRI (fMRI)* for brain function

- Planar radiography
  - mainly bone
  - projection

- MRI
  - bone and soft tissue
  - tomographic



Damage in cervical extensor muscle due to whiplash.

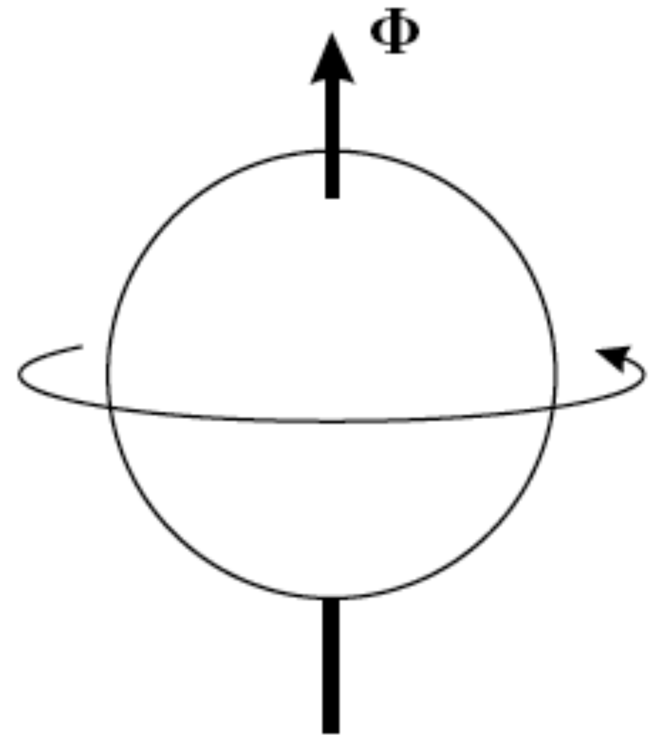
James Elliot
U. of Queenland
freshscience.org.au/?p=1463

Edema in the gastrocnemius (calf muscle) due to interstitial muscle tearing in “tennis leg”.

http://www.radsource.us/clinic/0608

Microscopic Magnetization

- NMR concerns nuclei, but *not* radioactivity
- Angular Momentum Φ
  - nuclei with *odd atomic number* or *odd mass number have non-zero quantum spin* I.
  - these have *spin*, and are NMR-active.
  - visualize as a small ball rotating on an axis.
  - "angular" means frequency is involved.
- *Nuclear spin systems*
  - collections of identical nuclei, regardless of their molecular environment (small changes)
  - ^{1}H, ^{13}C, ^{19}F, and ^{31}P are common nuclei, because they are isotopes of common biological elements.
  - ^{1}H is most common (water, fat) called "protons" (though actually other elements have protons too).



Non quantum physics description

- Each nucleus with positive charge spinning on an axis creates magnetic field with *magnetic moment* vector $\mu = \gamma \Phi$ angular momentum

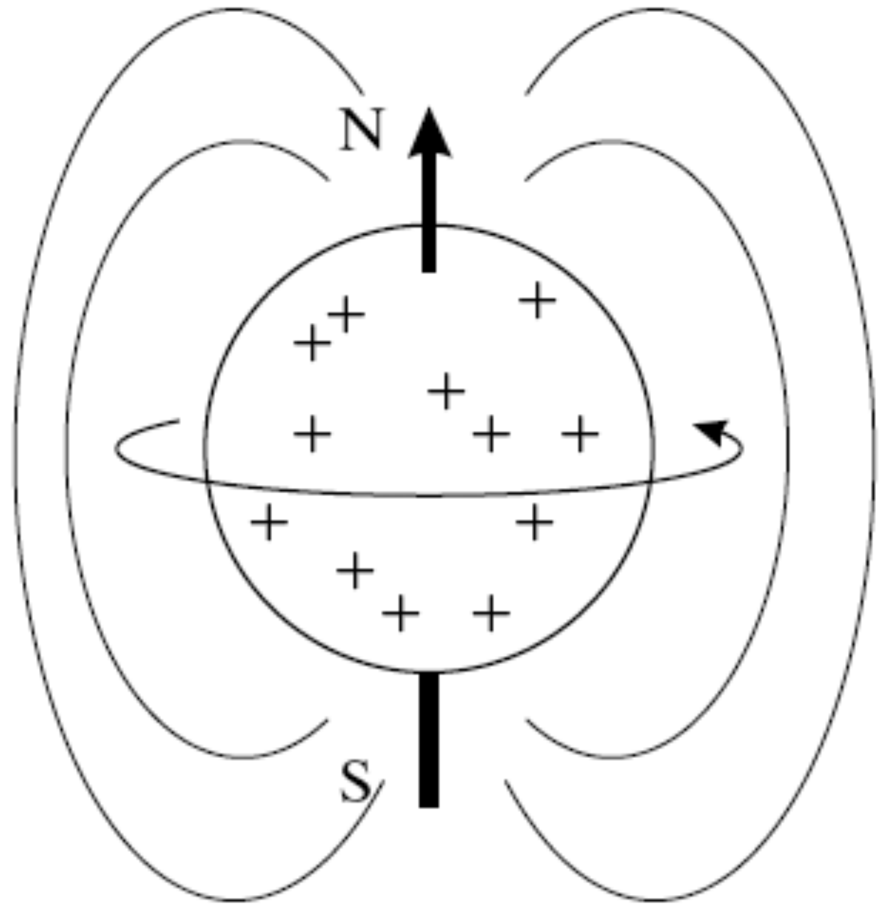
determines the torque experienced in an external magnetic field

γ is the *gyromagnetic ratio*

$$\gamma\!\!\!- = \frac{\gamma}{2\pi}$$

Common Gyromagnetic Ratios

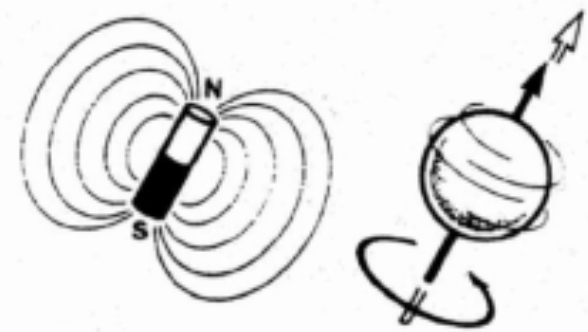
Nucleus	$\gamma\!\!\!-$ [MHz/T]
^{1}H	42.58
^{13}C	10.71
^{19}F	40.05
^{31}P	11.26



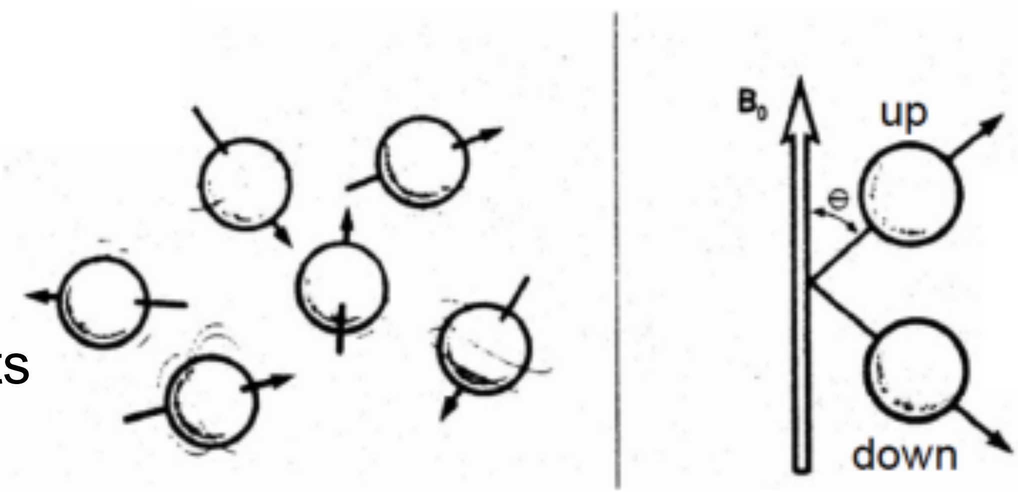
1 Tesla = 10^4 Gauss
Earth's magnetic field = 1/2 Gauss

External Magnetic Field

- By convention in the +*z* direction, $\boldsymbol{B}_0 = B_0\hat{z}$
- Microscopic spins don't just line up in the external field
  - Each nuclear species has a *spin quantum number*, I, and $2I+1$ possible states.



- I = *1/2* for ^{1}H -- a "*spin 1/2 system*"
  - Only two states are possible.
  - 54º off the +*z* and -*z* direction.
  - Random distribution between *up* and *down* orientations.
  - Slight preference for *up* yields *bulk* or *macroscopic magnetization* in *z* direction.
  - Bulk *magnetization vector* $\boldsymbol{M}$ of N_S individual nuclear moments

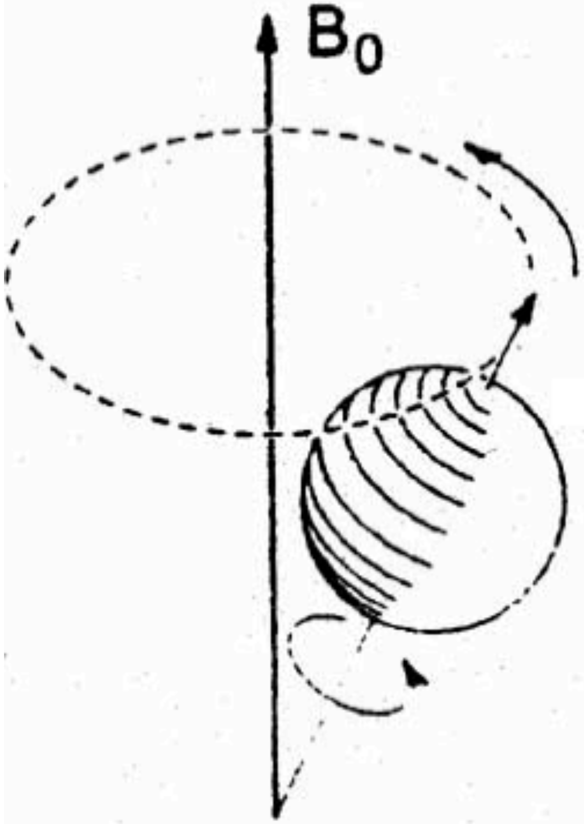


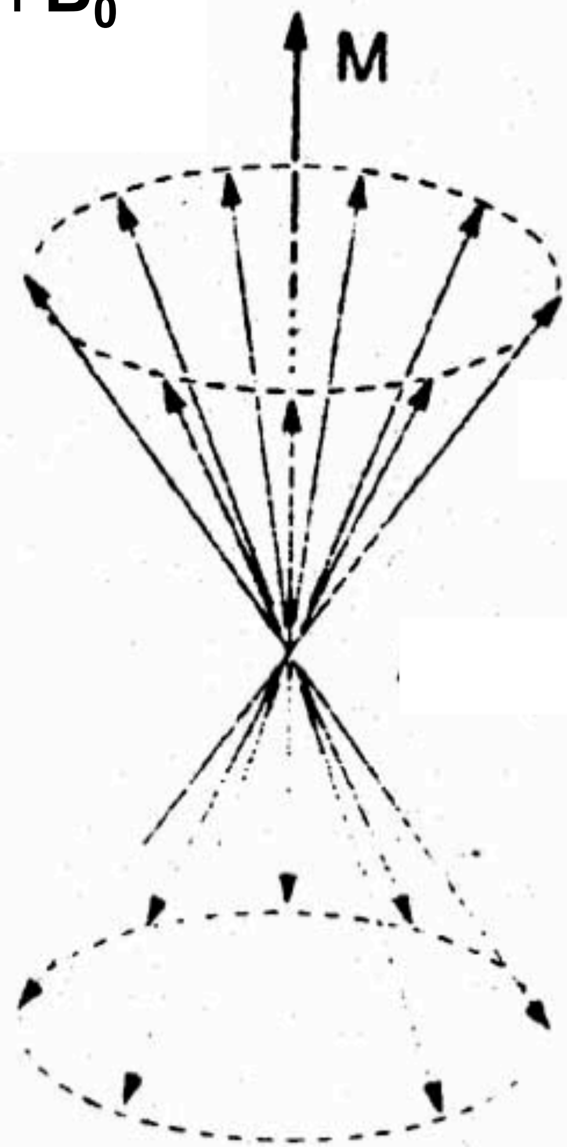
$$\boldsymbol{M} = \sum_{n=1}^{N_s} \boldsymbol{\mu}_n$$

⟵ these are vectors

Slight preference for *up* yields *bulk* or *macroscopic magnetization*

at equilibrium, aligned with $\mathbf{B_0}$





Each block of tissue generates M_0 net field.

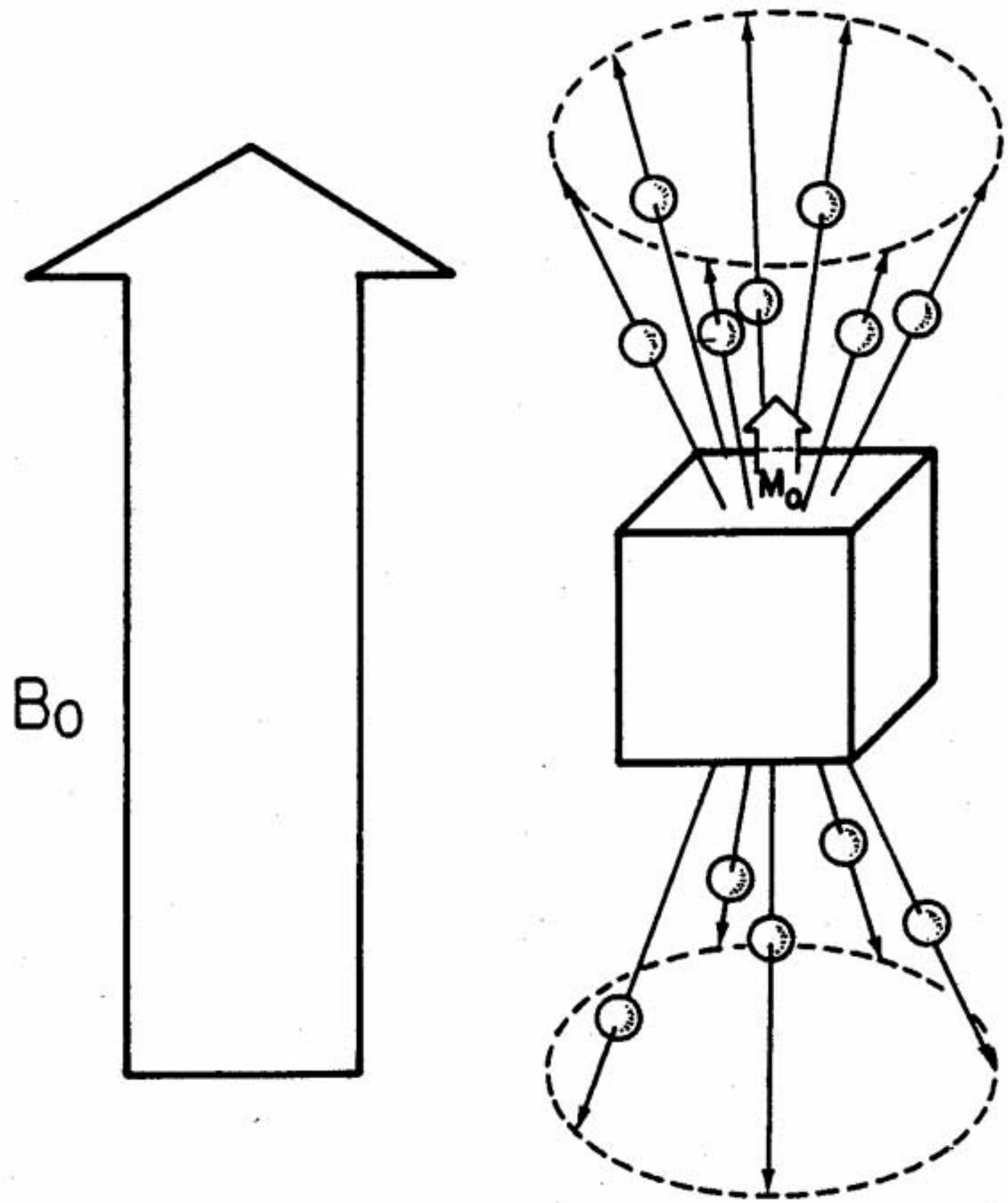


Figure 5. The net magnetization (M_0) resulting from the imbalance of hydrogen nuclear dipoles points along B_0 but is small compared with B_0.

Equilibrium Magnetization Vector $\boldsymbol{M}_0$

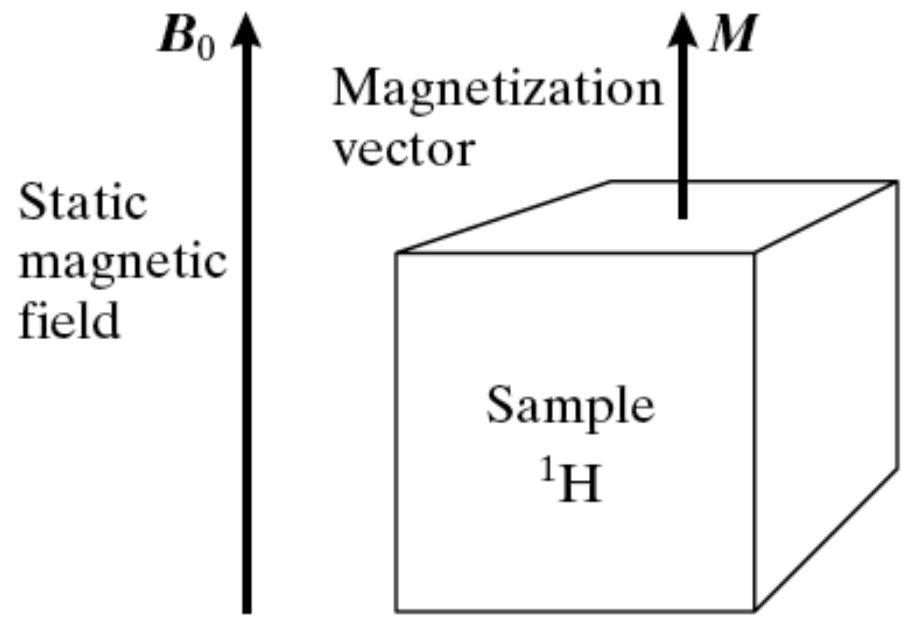
- After a long enough time, the magnitude of $\boldsymbol{M}_0$ at location $\boldsymbol{r} = (x, y, z)$ is

Planck's constant

$$M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D$$

Proton density

Boltzmann's constant



The NMR signal will be proportional to M_0 (once tipped over and spinning). Thus larger magnets and higher proton densities lead to larger signals

Angular Momentum J - Magnetization Vector M

- For each voxel (sample) at each point in time, there is a *magnetization vector* $\mathbf{M} = \mathbf{M}(\mathbf{r}, t)$ such that

$$\mathbf{M} = \gamma \mathbf{J}$$

angular momentum

the macroscopic version of microscopic magnetic moment,

$$\boldsymbol{\mu} = \gamma \boldsymbol{\Phi}$$

A torque results between the magnetic moment and the external magnetic field

$$\frac{d\mathbf{J}(t)}{dt} = \mathbf{M}(t) \times \mathbf{B}(t)$$

by substitution yields the 1rst order vectoral differential equation

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)$$

Precession - like a spinning top in gravity

- Assuming a static magnetic field $B(t) = B_0$, but $\boldsymbol{M}$ not necessarily aligned with $\boldsymbol{B}_0$, the solution is

torque makes it spin.

$$M_x(t) = M_0 \sin\alpha \cos(-\gamma B_0 t + \phi),$$

tip angle

$$M_y(t) = M_0 \sin\alpha \sin(-\gamma B_0 t + \phi),$$

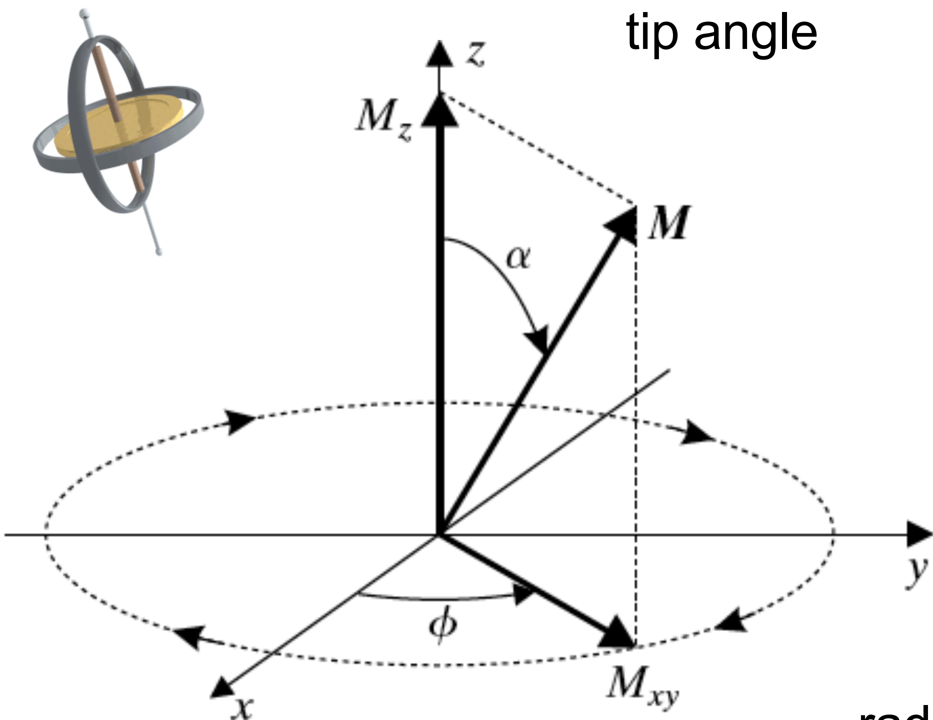
$$M_z(t) = M_0 \cos\alpha \,,$$

tip angle

Where

ϕ is an arbitrary angle

and $M_0 = |\boldsymbol{M}(0)|$



Precession of magnetization vector

$$\boldsymbol{M}(t) = (M_x(t), M_y(t), M_z(t))$$

around $\boldsymbol{B}_0$ at the *Larmor frequency*

$$\omega_0 = \gamma B_0 \qquad\qquad \nu_0 = \bar{\gamma} B_0$$

radians per second cycles per second

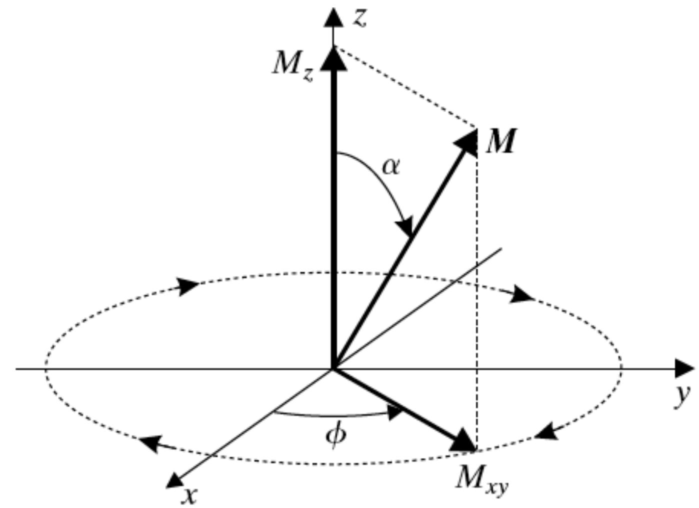
Precession is at the *Larmor Frequency*

- Substituting $\nu_0 = \gamma B_0$

$$M_x(t) = M_0 \sin\alpha \cos(-2\pi\nu_0 t + \phi)$$

$$M_y(t) = M_0 \sin\alpha \sin(-2\pi\nu_0 t + \phi)$$

$$M_z(t) = M_0 \cos\alpha \; .$$



- Three sources of fluctuation in $\boldsymbol{B}_0$ and hence Larmor freq.
  - *Magnetic field inhomogeneity*: corrected by *shimming* to make it very homogeneous (a few parts per million).
  - *Magnetic susceptibility*: material properties that decrease or increase field within a material relative to the surrounding field. All materials are *diamagnetic* and slightly lower the field, other materials are also *paramagnetic* or *ferromagnetic* and increase the field.
  - *Chemical shift*: H shielded by electron clouds in particular associated molecule within a given *isochromat* (ς = -3.35 ppm for fat vs. water)

$$\hat{B}_0 = B_0(1 - \varsigma) \longrightarrow \hat{\nu}_0 = \nu_0(1 - \varsigma)$$

shielding constant (stigma)

Transverse and Longitudinal Magnetization

- Longitudinal is $M_z(t)$—simply the z-component of $\mathbf{M}(t)$

- Transverse magnetization $M_{xy}(t) = M_x(t) + jM_y(t)$
  - Incorporates $M_x(t)$ and $M_y(t)$ into one *complex number,* with *phase*

$$\phi = \tan^{-1}\frac{M_y}{M_x}$$ (when t = 0)

Thus

tip angle

$$M_x(t) = M_0 \sin\alpha \cos(-2\pi\nu_0 t + \phi)$$

$$M_y(t) = M_0 \sin\alpha \sin(-2\pi\nu_0 t + \phi)$$

becomes

$$M_{xy}(t) = M_0 \sin\alpha\, e^{-j(2\pi\nu_0 t - \phi)}$$

a real parameter with a complex value!

NMR Signals

- Transverse magnetization creates RF excitation.
  - Will induce a voltage in a coil of wire outside the sample.
  - Signal is not detected from *radio waves*, but rather is induced at (as in a generator) at a *radio frequency*.
  - Does *not* contribute an irradiation dose to the patient.
- Faraday's law of induction - *Principle of reciprocity*
  - Suppose the magnetic field produced at location $\boldsymbol{r}$ by a <u>unit</u> direct current in a coil would be $\boldsymbol{B}^r(r)$
  - Now reverse the scenario, with a time-varying magnetic field

$$V(t) = -\frac{\partial}{\partial t}\int_{\text{object}} \mathbf{M}(r,t)\cdot \boldsymbol{B}^r(r)\, dr$$

voltage induced in coil

derivative in time from time-varying *M(r,t)*

dot product

integrated in space

Induced Voltage in NMR Sample

- Assume
  - homogeneous sample, $M(r,t) = M(t)$
  - uniform field produced by coil, $B^r(r) = B^r$
  - The *z* component of magnetization is only slowly changing (ignore its derivative, leaving just transverse magnetization).

$$V(t) = -\frac{\partial}{\partial t}\int_{\text{object}} \overbrace{M_x(t)B_x^r + M_y(t)B_y^r}^{\text{dot product expanded}}\, d\mathbf{r}\,,$$

$$= -V_s \frac{\partial}{\partial t}\left[M_x(t)B_x^r + M_y(t)B_y^r\right],$$

where V_s is the volume of the sample.

and

B^r is in *x*-*y* plane with angle θ

$$B_x^r = B^r \cos\theta_r$$

$$B_y^r = B^r \sin\theta_r$$

Induced Voltage in NMR Sample

- After some trigonometric manipulation

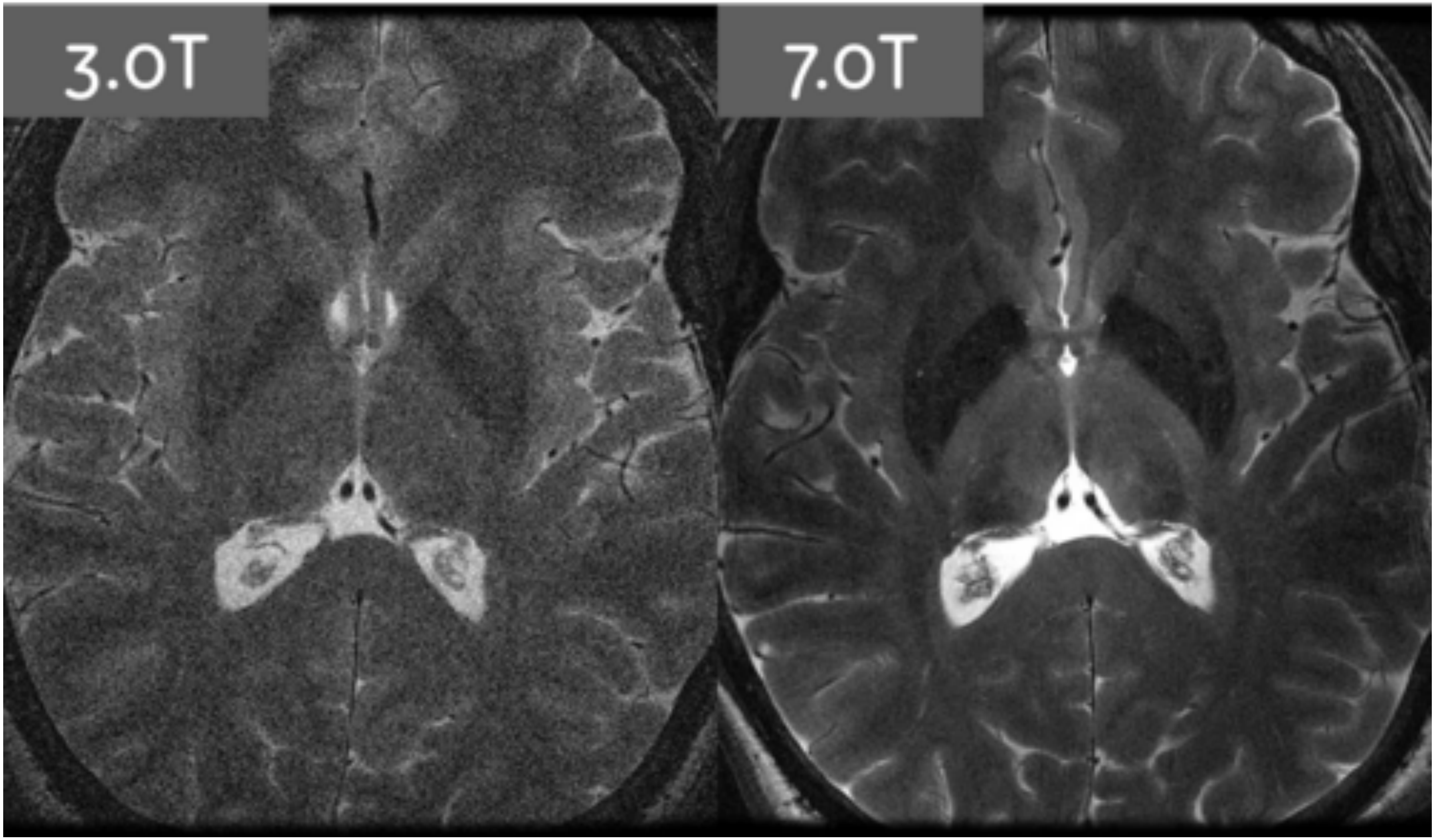
$$V(t) = -2\pi \nu_0 V_s M_0 \sin \alpha \, B^r \sin(-2\pi \nu_0 t + \phi - \theta_r)$$

tip angle

$$|V| = 2\pi \nu_0 V_s M_0 \sin \alpha \, B^r$$

- Since $\nu_0 = \gamma B_0$, signal strength is proportional to B_0^2
  - …since M_0 is also proportional to B_0.
  - Stronger magnet gives stronger signal.
- Maximum signal when *tip angle is 90 degrees* $\alpha = \pi/2$
  - Smaller tip angles can, however, be created faster, as will be seen.
- Larger samples (bigger V_s) gives stronger signal
  - However, this is at the expense of spatial resolution.

7T vs 3T signal-to-noise (translates into resolution)



http://www.ge.com/reports/post/85638328370/these-magnets-are-140000-times-stronger-than/

Rotating Frame

- General form for rotation by angle a
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
- We create a reference frame rotating at the Larmor frequency

$$x' = x\cos(2\pi\nu_0 t) - y\sin(2\pi\nu_0 t),$$

$$y' = x\sin(2\pi\nu_0 t) + y\cos(2\pi\nu_0 t),$$

$$z' = z.$$

in which $M_{xy}(t) = M_0 \sin\alpha e^{-j(2\pi\nu_0 t - \phi)}$

becomes $M_{x'y'}(t) = M_0 \sin\alpha e^{j\phi}$

$M_{x'y'}$ is a *stationary vector* in the rotating complex plane with magnitude $M_0 \sin\alpha$ and phase angle ϕ.

$$M_{x'y'} = M_{xy} e^{j2\pi\nu_0 t}$$

RF pulse turns M_0 from *z* axis down into *x-y* plane (M_{XY}), where it acts as a little RF generator.

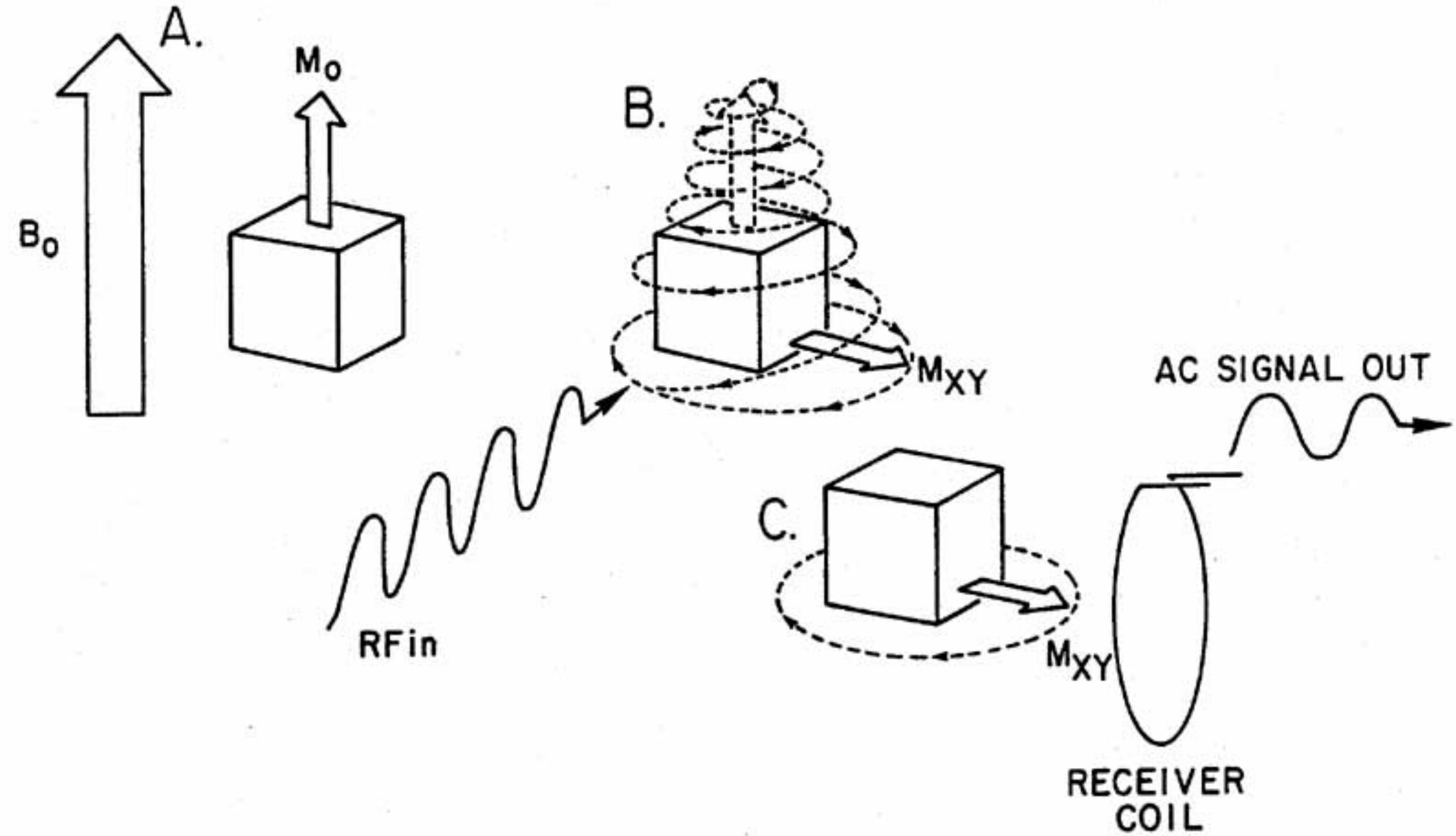


Figure 6. *A*, Net magnetization of the sample M_0 initially is aligned with the main magnetic field B_0, but it is so small in comparison to B_0 that it is undetectable. *B*, A radio-frequency (RF) field applied at the Larmor frequency tips tissue magnetization into the transverse plane, rendering it measurable as transverse magnetization, M_{xy}. *C*, Measurement of M_{xy} is possible because of its precession, which produces a changing magnetic flux linking a properly oriented loop receiver coil. The changing magnetic flux linking the coil induces an alternating current (AC) (alternating at the Larmor frequency) in the receiver coil. This alternating current, when amplified and digitized, becomes the signal from which the MR image is reconstructed.

RF Excitation

- In a system at equilibrium $M(t)$ lines up with B_0
- If a small magnetic field $B_1 = B_1\hat{x}$ is turned on, then

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)$$

predicts a small motion of $\boldsymbol{M}(t)$ in the +*y* direction
a *precession* around the *x*-axis.

- To continually push $\boldsymbol{M}(t)$ towards the transverse plane, we can apply the $B_1 = B_1\hat{x}$ field at the Larmor frequency, so that $\boldsymbol{M}(t)$ is pushed down when it coincides with the y-axis (*linearly polarized*). Alternatively, we can add a *y*-component to $\boldsymbol{B}_1$ so that it can push $\boldsymbol{M}(t)$ towards the transverse plane continually (*circularly polarized*).

Circularly polarized

- *Quadrature* (sin and cos) RF coils produce circularly polarized RF field, modeled as complex in the transverse plane.

complex envelope　　　initial phase

$$B_1(t) = B_1^e(t)e^{-j(2\pi\nu_0 t-\varphi)}$$

- If envelope is simple rectangular pulse, then in *rotating frame*

$$B_1(t) = B_1^e(t)e^{j\varphi}$$

assume 0, so B_1 oriented in x' direction

- *Forced precession* in the $y'z'$ plane, due to RF field

different Larmor frequency, slower than ν_0

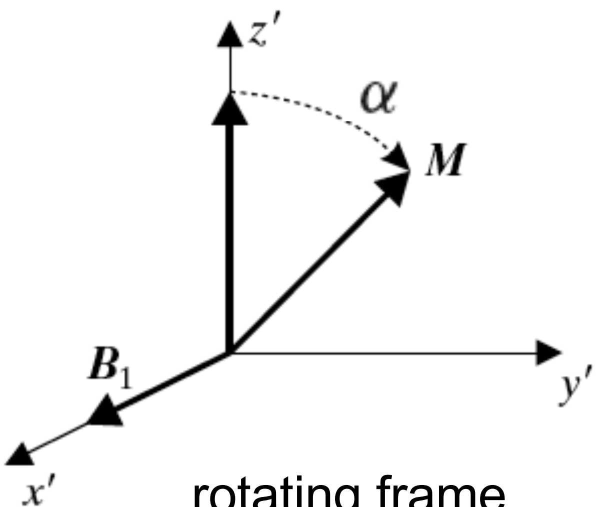
$$\nu_1 = \gamma B_1, \text{ where } B_1 = |B_1^e(t)|$$

- Integral of Excitation Pulse over *pulse duration* determines *tip angle*

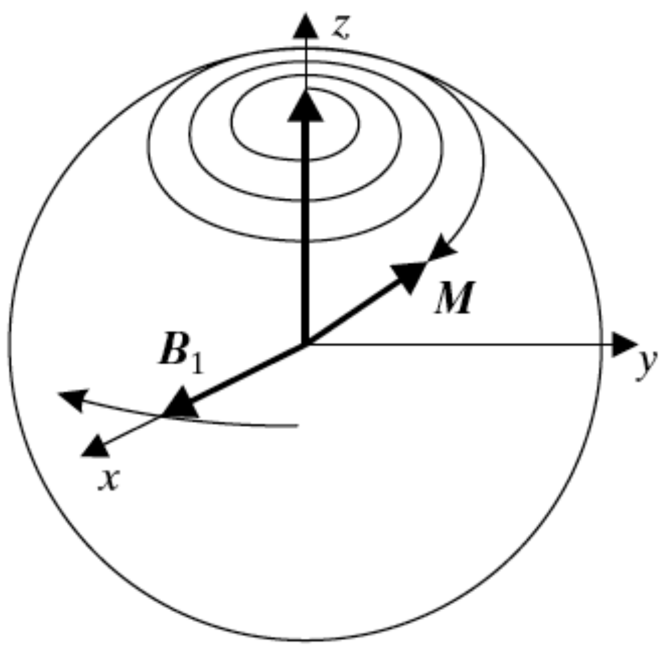
$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

- For rectangular pulse, *tip angle* determined by *pulse duration*

$$\alpha = \gamma B_1 \tau_p$$



rotating frame



stationary frame

$\alpha = \pi/2$ is common, yields max signal

When $\alpha = \pi$ it is called an *inversion pulse*

Relaxation

- After an *excitation pulse* (or α-*pulse*) the transverse magnetization generates an RF signal, but only for a while (otherwise, that would constitute perpetual motion).
- There are two main mechanisms by which the RF signal disappears:

Transverse Relaxation: de-phasing of the magnetization vectors within the sample.

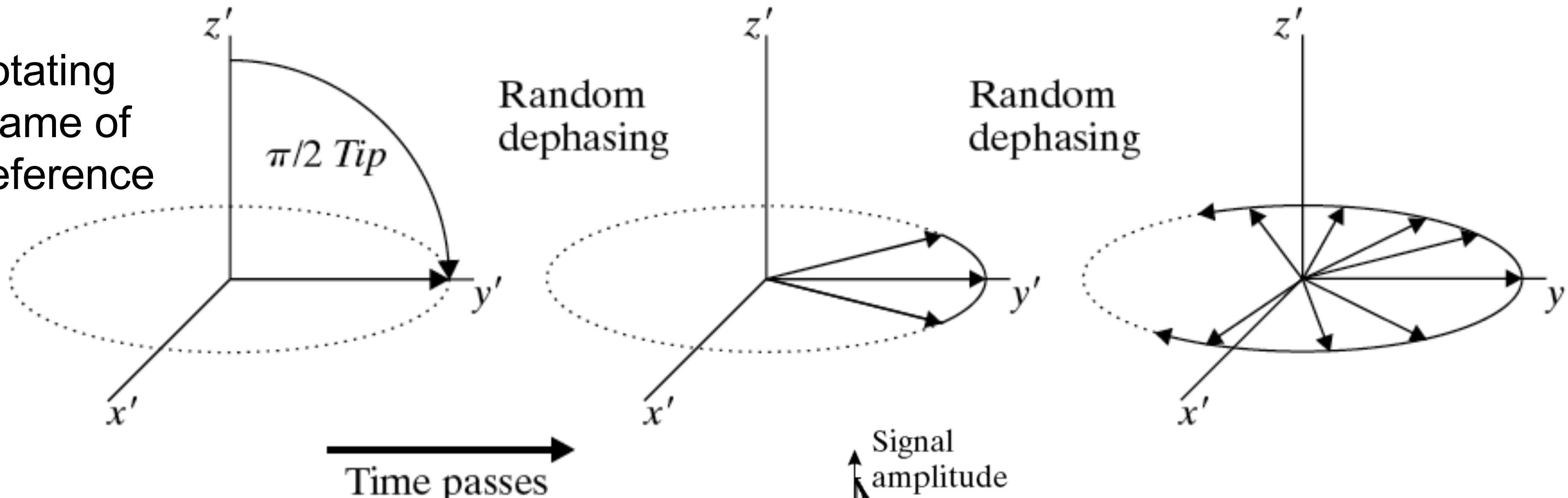
Longitudinal Relaxation: re-establishment of equilibrium with the magnetization vector aligned with $\boldsymbol{B}_0$

- These happen at particular rates for particular tissues, and account for most of the contrast in MRI.

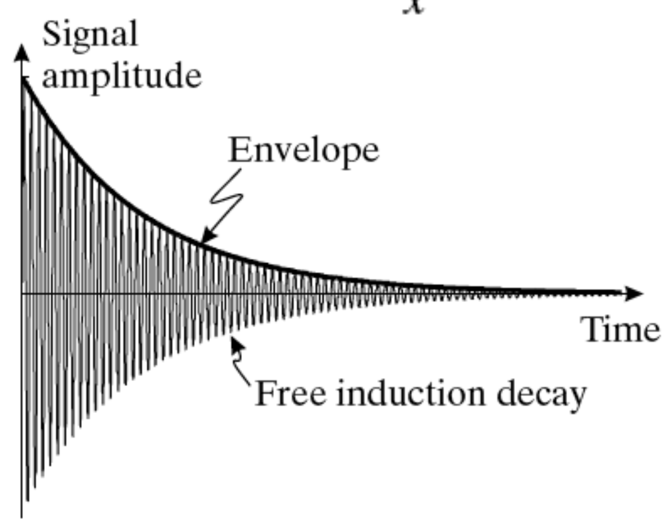
Transverse Relaxation

- Also called *Spin-Spin Relaxation*: due to randomly varying perturbations of the magnetic field due to other spins nearby.
- Different magnetic fields cause different frequencies and dephasing of transverse magnetization within the sample.

rotating
frame of
reference



- Loss of coherence causes the RF signal to exhibit *free induction decay* (FID)

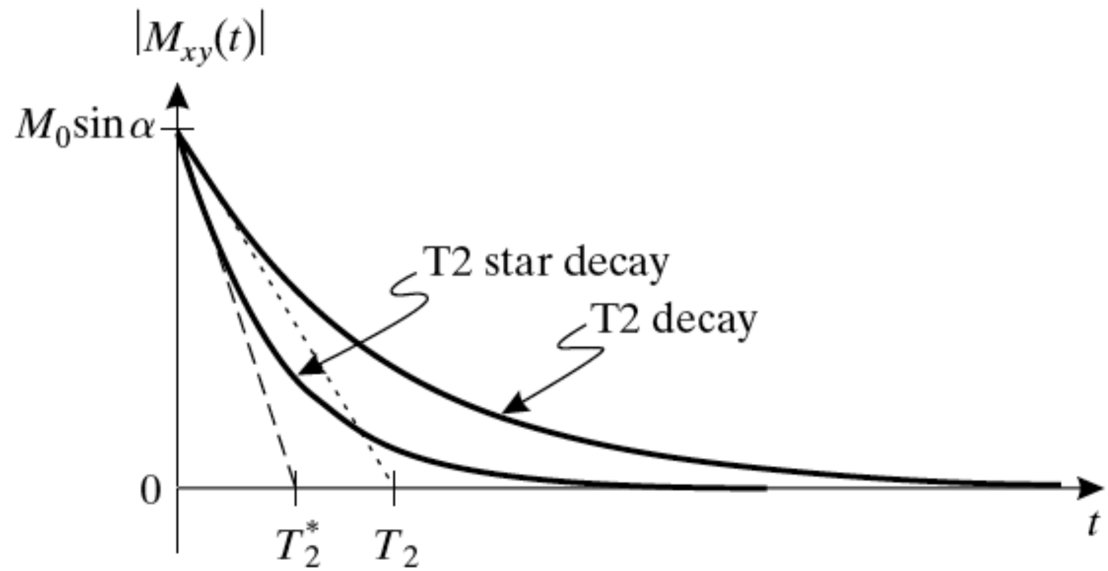


- Modeled well as exponential decay with a tissue-dependent time constant T_2, the *transverse relaxation time*.

$$\boxed{M_{xy}(t) = M_0 \sin\alpha \, e^{-j(2\pi\nu_0 t - \phi)} e^{-t/T_2}}$$

- The actual transverse relaxation time T_2* ("tee two star") is shorter, due to additional non-varying inhomogeneity in $\boldsymbol{B}_0$.

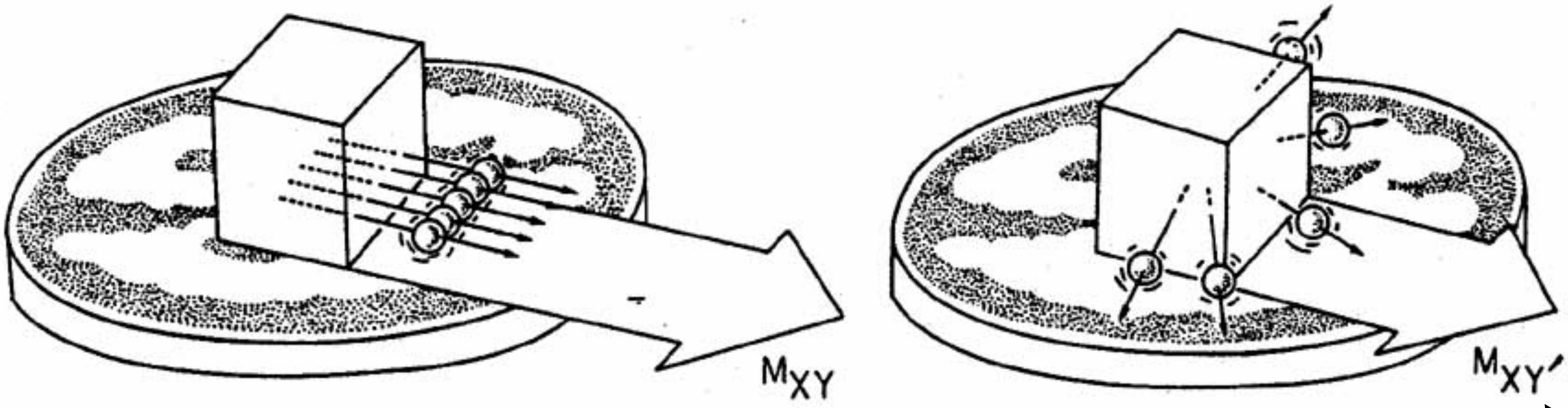
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$



- The non-varying (constant) inhomogeneity (which is reversible, as we shall see) would by itself cause dephasing with a time constant T_2' ("tee two prime").

M_{XY} dephases with time constant T_2^* and corresponding loss of RF signal.

(shown in rotating frame)



Notation (') from other book to mean M attenuated by dephasing. Prince says $M_{x'y'}$ to mean M in the rotating frame of reference.

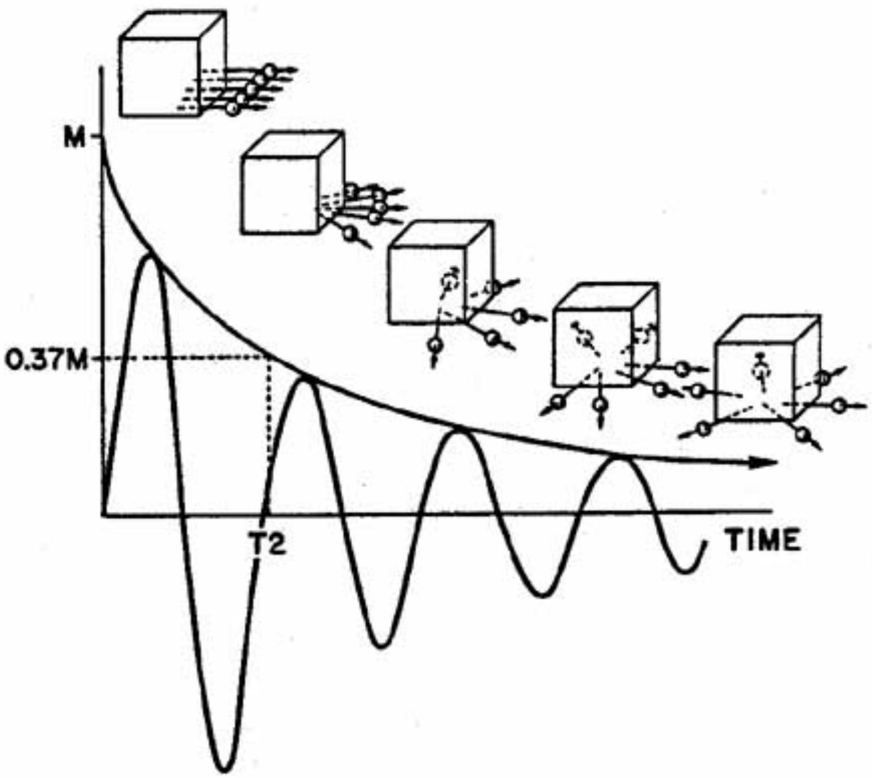
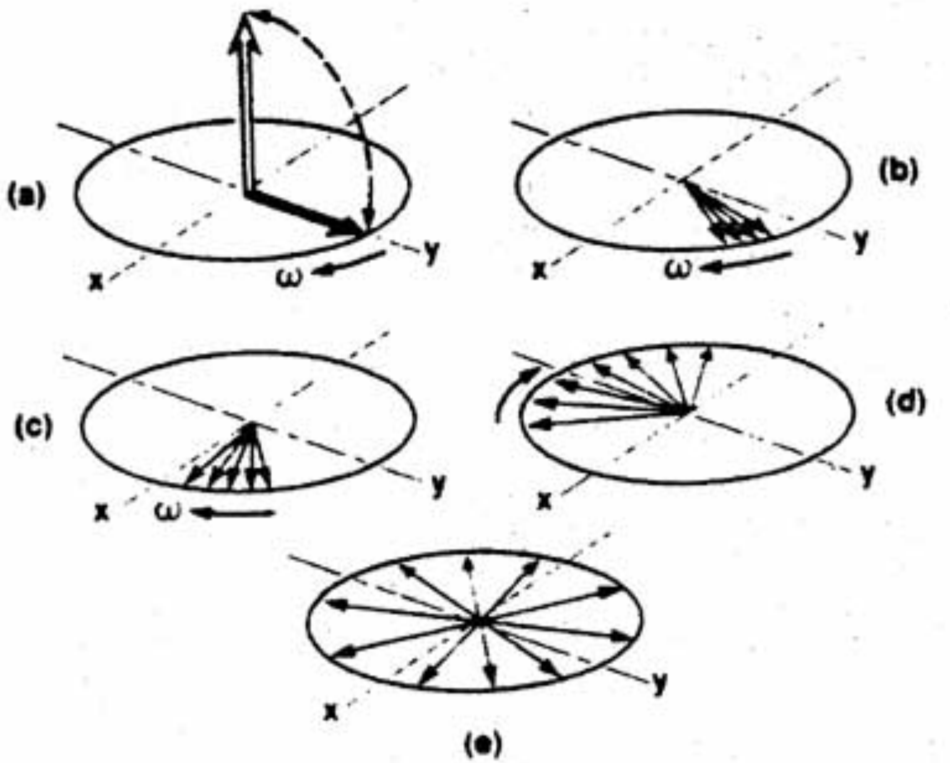
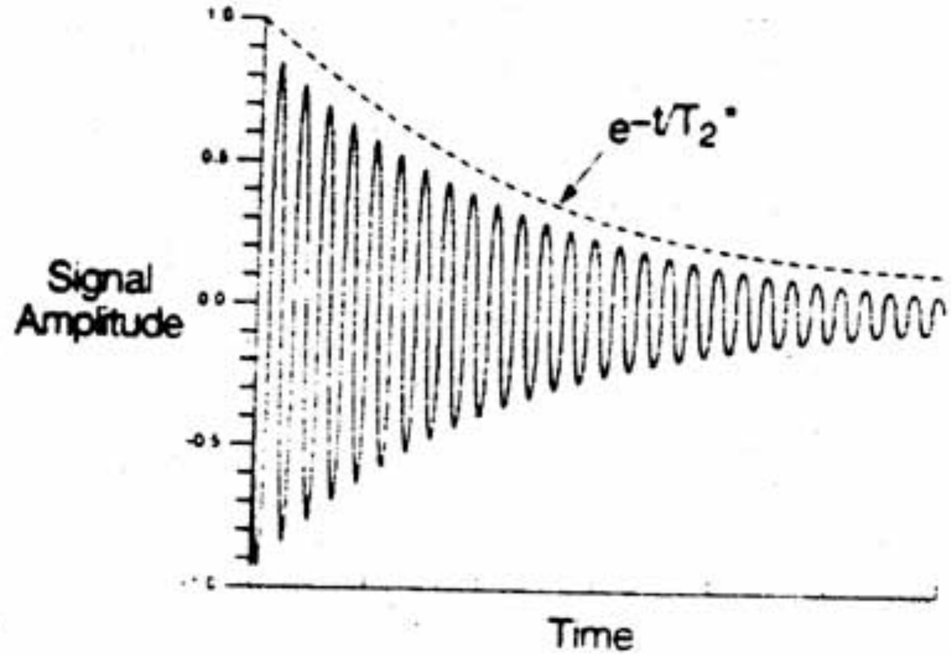


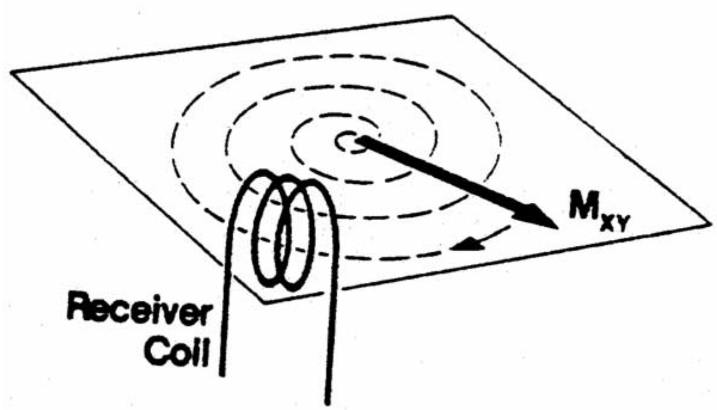
Figure 8. (**a**) Immediately after a 90° RF pulse, the magnetic dipoles of individual nuclei are precessing in phase, and the transverse magnetization vector, M_{xy}, is maximal. (**b**) As time progresses, magnetic dipoles lose phase coherence, some precessing faster and some slower, due to the local magnetic environment. This loss of phase coherence causes a decrease in the net transverse magnetization, with M_{xy}' less than M_{xy}. (**c**) As a result, the signal recorded by the receiver coil decreases exponentially in amplitude. T2 is defined as the time required for the transverse magnetization to decay to 37% of its original level.

Dephasing with time constant T_2*

(shown in stationary frame)







Longitudinal Relaxation

- Also called *Spin-Lattice Relaxation*: due to interactions with neighboring atoms that lead back to equilibrium, with the magnetization vector aligned with $\boldsymbol{B}_0$
- Modeled well as exponential decay with a tissue-dependent time constant T_1, the *longitudinal relaxation time*.

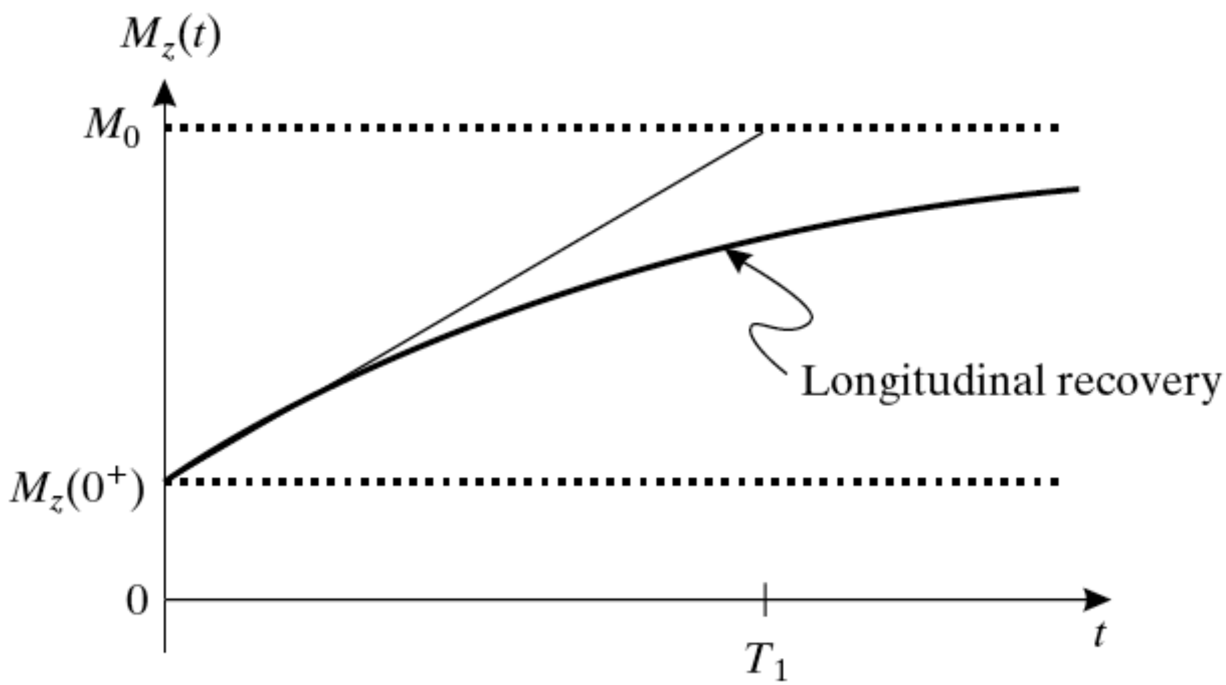
$$\boxed{M_z(t) = M_0(1 - e^{-t/T_1})} + M_z(0^+)e^{-t/T_1}$$

if $\propto = 90°$

$$M_z(0^+) = M_0 \cos\alpha$$

α-pulse is applied at $t = 0$.

0^+ means just after t = 0



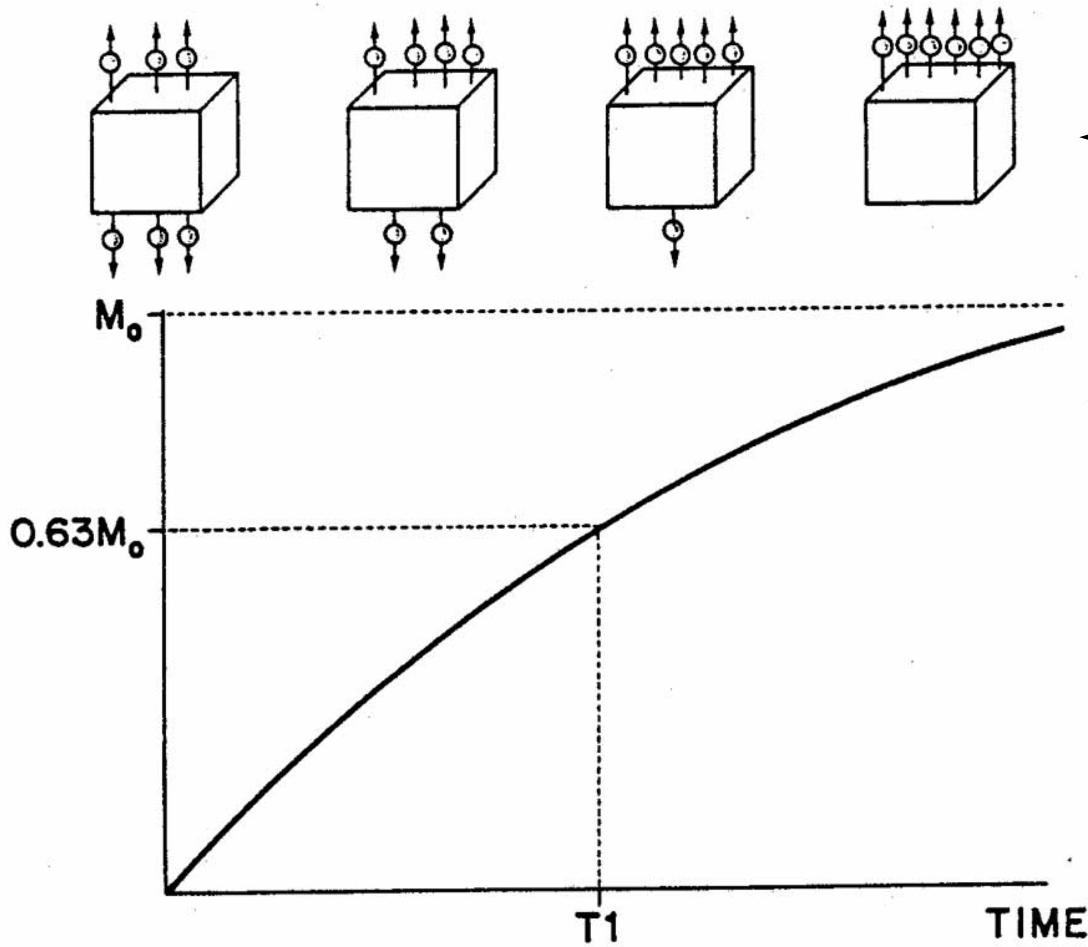


Figure 7. T1 recovery as a function of TR after a 90° pulse. Immediately after the 90° pulse, the population of higher-energy dipoles (antiparallel to B_0, pointing downward) and lower-energy dipoles (parallel to B_0, pointing upward) is equal. As energy is transferred from excited, higher-energy dipoles to the surrounding macromolecules, the longitudinal magnetization approaches its equilibrium value, M_0, which is a maximum imbalance of dipoles. T1 for a given tissue is defined as the time delay required after a 90° pulse for 63% of the tissue magnetization to recover along the direction of B_0.

Never actually all point up

M_0 is regenerated with time constant T_1 due to spin-lattice interactions.

T_1 (longitudinal) and T_2 (transverse) relaxation times

- In general, T_2 much shorter than T_1

  $250 \text{ ms} < T_1 < 2500 \text{ ms}$

  $25 \text{ ms} < T_2 < 250 \text{ ms}$

- The sample is said to be at *equilibrium* after $3T_1^{max}$, the longest T_1 in the sample.
- As we shall see, the term *steady-state* will be used to mean something different, when periodic excitation (RF) pulses are occurring at intervals shorter than $3T_1^{max}$

The Bloch Equations

- Combining the *forced* and *relaxation* behavior of a magnetic spin system yields a matrix 1rst order differential equation(s)

precession relaxation

$$\frac{dM(t)}{dt} = \gamma M(t) \times B(t) - \mathrm{R}\{M(t) - M_0\}$$

where $B(t) = B_0 + B_1(t)$ is composed of the static and RF fields, and where the matrix R is multiplied by the magnetic vector

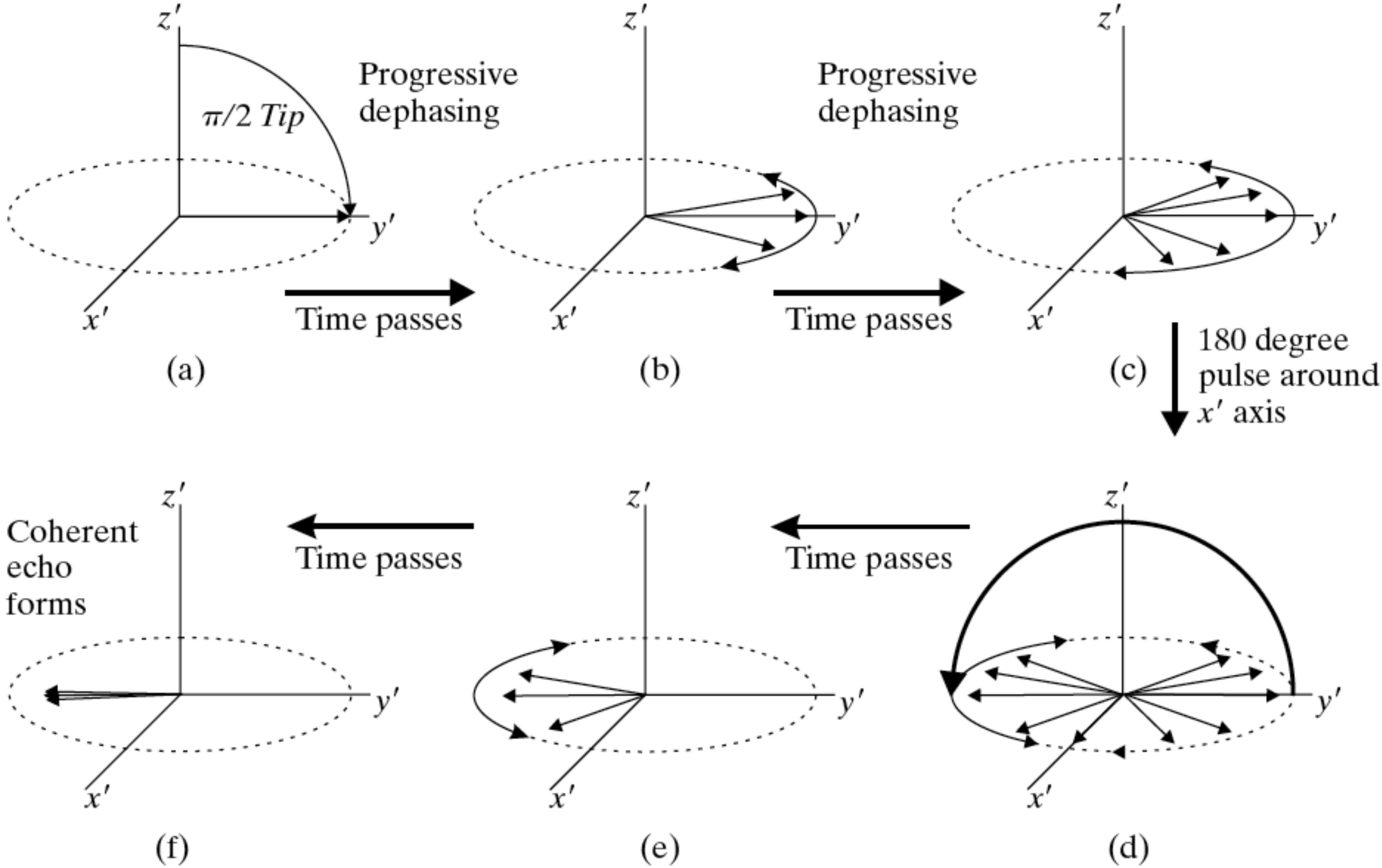
$$\begin{pmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{pmatrix} \begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix}$$

Produces the exponential equations.

to produce all the right expanded terms we have already discussed.

(see Example 12.5)

Spins are "flipped" with a 180º (π) pulse



not perfectly together because of tissue-dependent T_2 dephasing

Spin Echoes

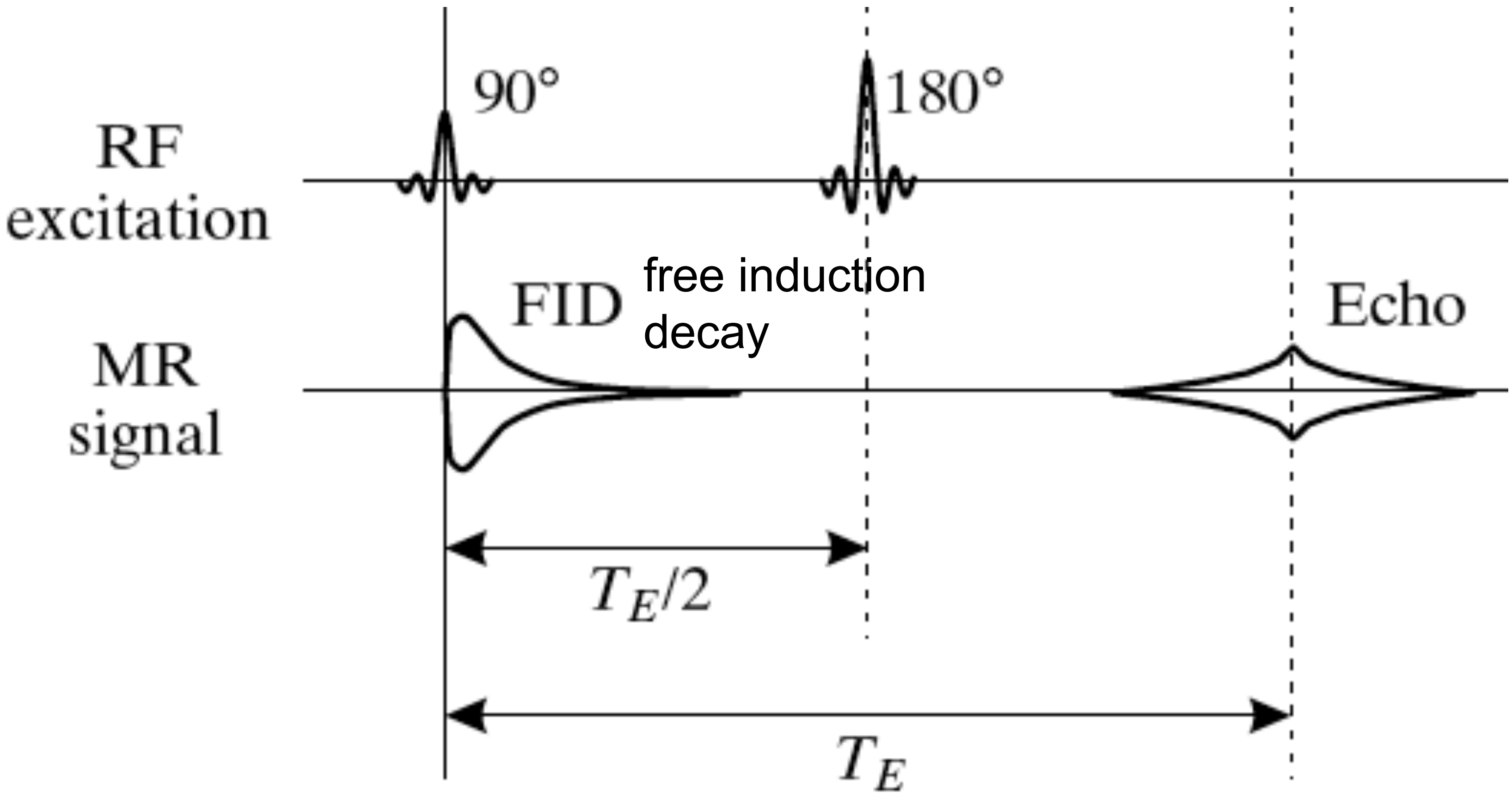
- T_2* is made up of a desired tissue-dependent T_2 and a undesired T_2', such that

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

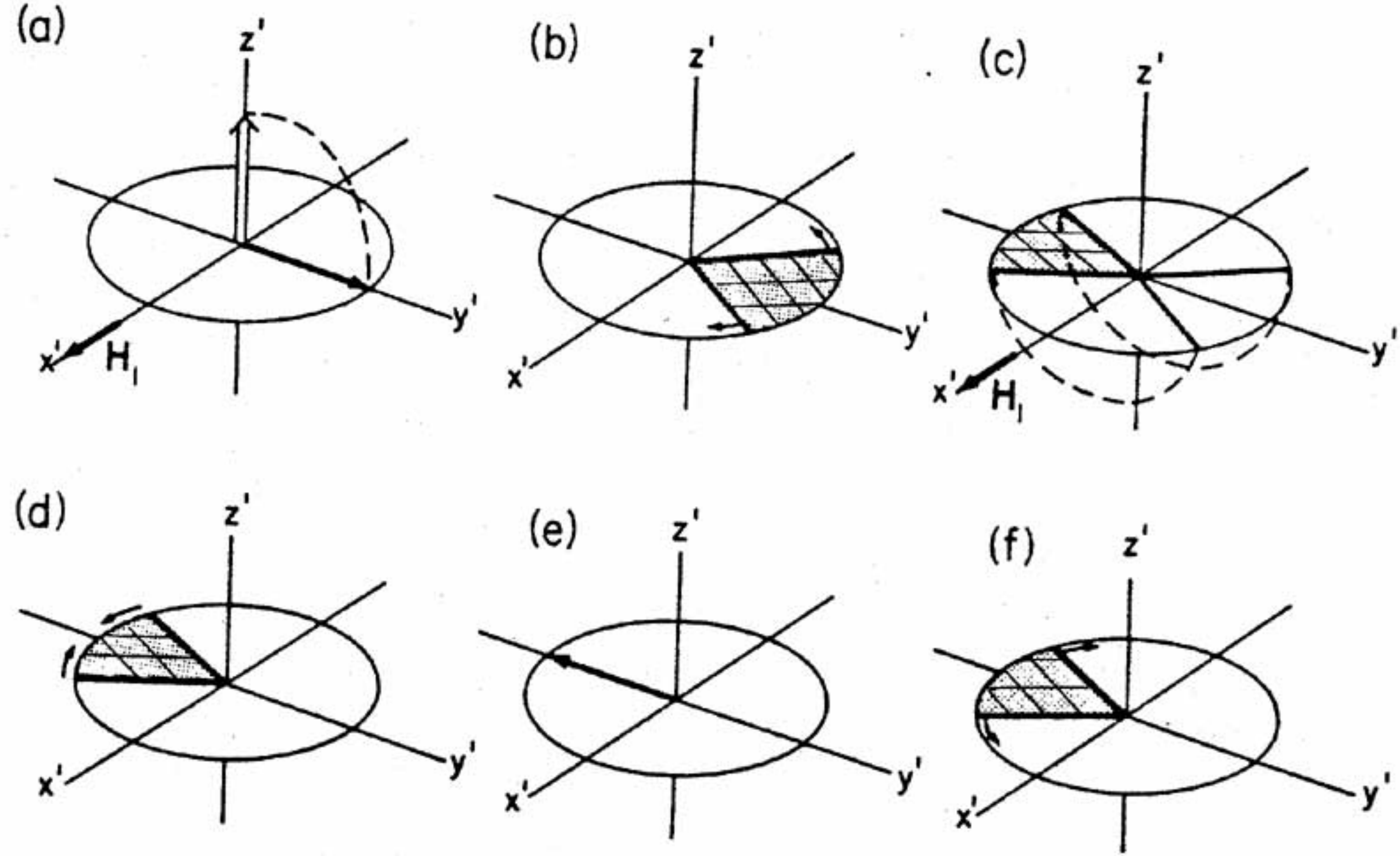
- T_2 is due to rapidly varying local conditions and truly random.
- T_2' however is due to fixed perturbations in the magnetic field, and therefore is reversible.
  - Local spins that are faster will gain ground over a period of time ($T_E/2$, where T_E is the "echo time").
  - If those local spins are then reversed, they will give back the extra ground over the same amount of additional time ($T_E/2$).
- The strength of the resulting "echo" at time T_E will be due solely to T_2, the desired tissue-dependent parameter.

Spin Echo pulse sequence

- Strength of echo now due to ~~2 factors~~ T_2
  - ~~*Longitudinal relaxation* due to *spin-lattice* interactions converting some of the *transverse magnetization* back into *longitudinal magnetization* (note how the arrows in the previous slide at (f) are somewhat shorter).~~
  - *Transverse relaxation* due now to the tissue dependent *spin-spin* interactions determining T_2 without the undesired effects from T_2'.

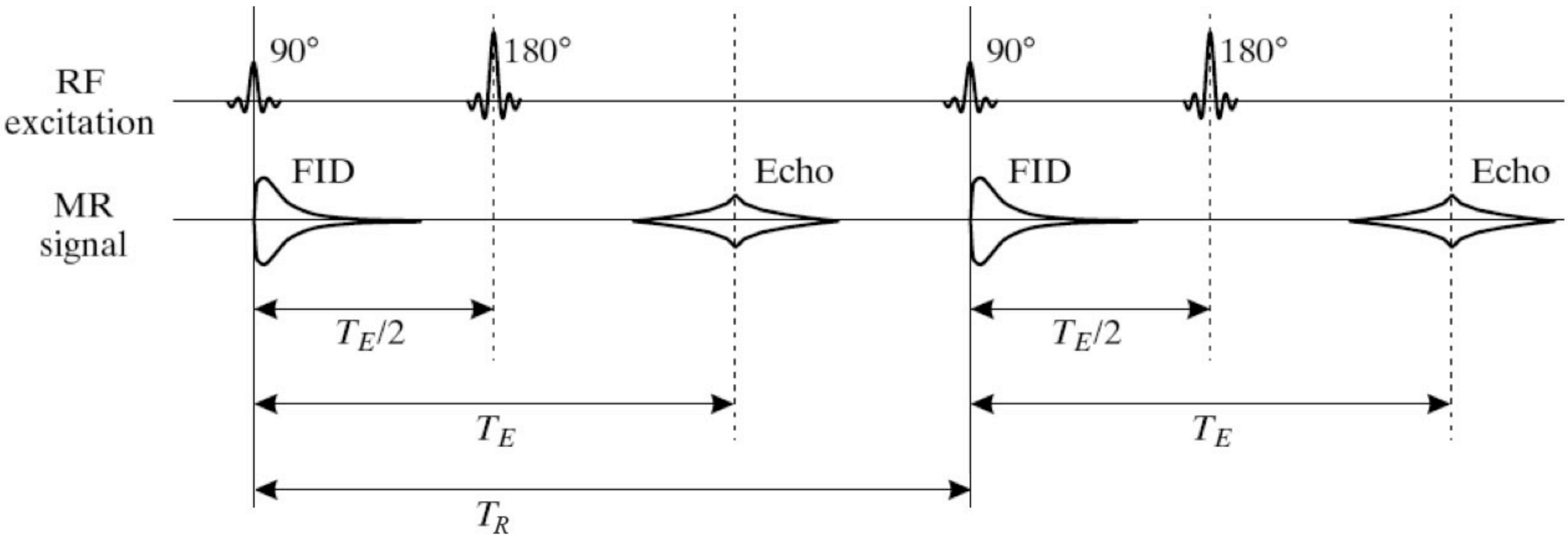


180^o RF pulse flips protons so that they rephase



Multiple echoes can be elicited by multiple180^o RF.

T_R is the time between one $\pi/2$ RF pulse and the next

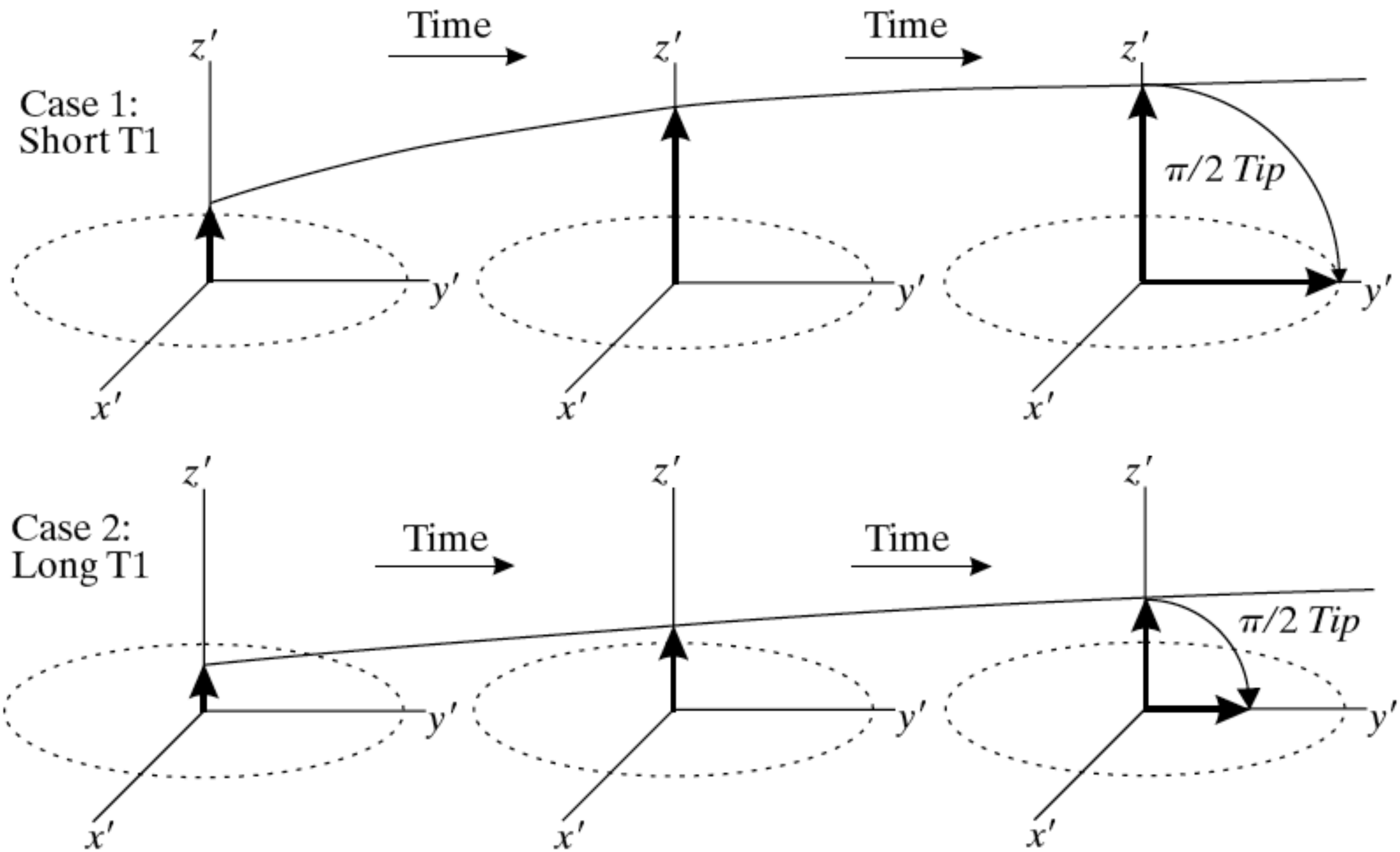


T_R, the *pulse repetition interval*, determines if a tissue has enough time to re-form its longitudinal magnetization, given its T_1.

Only longitudinal magnetization is available to be tipped next time, and thereby determines the strength of the next signal.

T_1 - weighted contrast

- Longitudinal magnetization available to flip approaches *steady state* after several repetition intervals, which depends on T_1

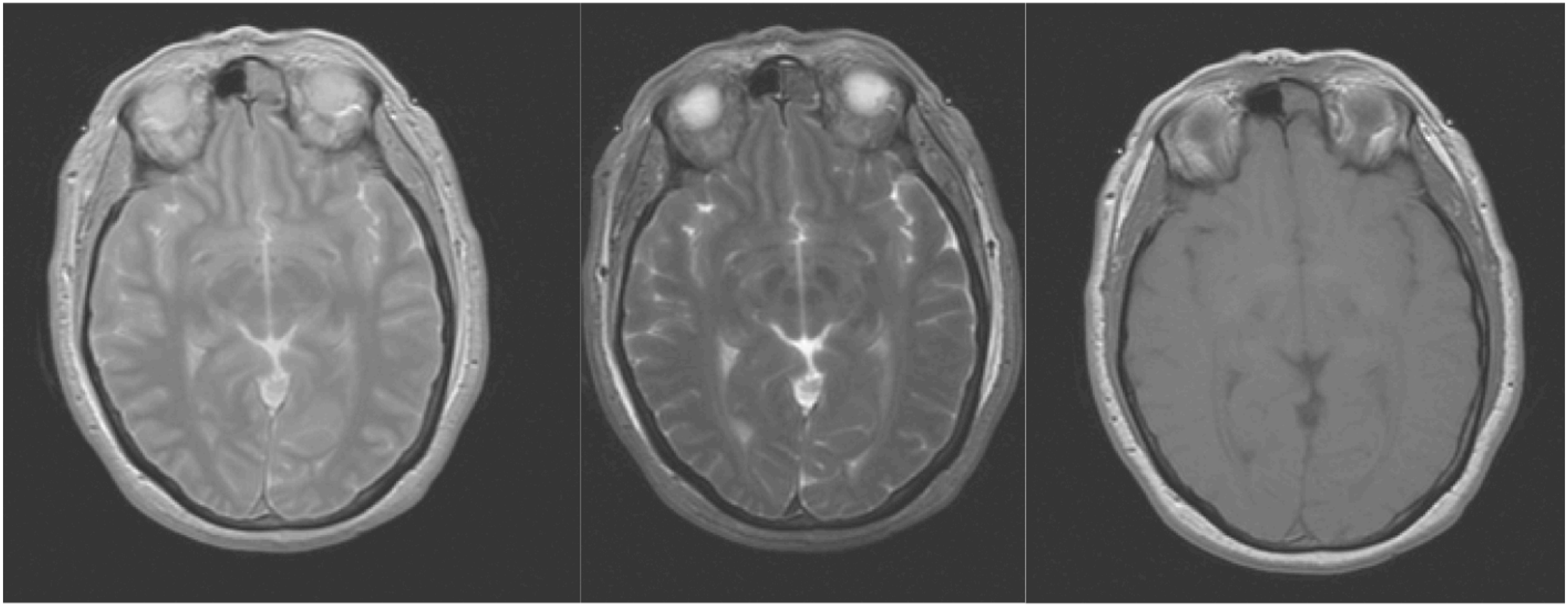


Contrast Mechanisms

- 3 intrinsic tissue properties
  - P_D *proton density* (number of hydrogen atoms per unit volume).
  - T_2 *transverse relaxation time*
  - T_1 *longitudinal relaxation time*
- 3 main control parameters
  - α *tip angle*, the strength and duration of the RF pulse
  - T_E *echo time*, how long before echo is recorded
  - T_R *pulse repetition interval*, how long between successive α-pulses
- 3 common spin echo image types
  - P_D - weighted
  - T_2 - weighted
  - T_1 - weighted

Does not mean that the pixel brightness is proportional to these parameters, but merely that contrast is due primarily to differences in these parameters respectively.

3 common spin-echo image contrast settings



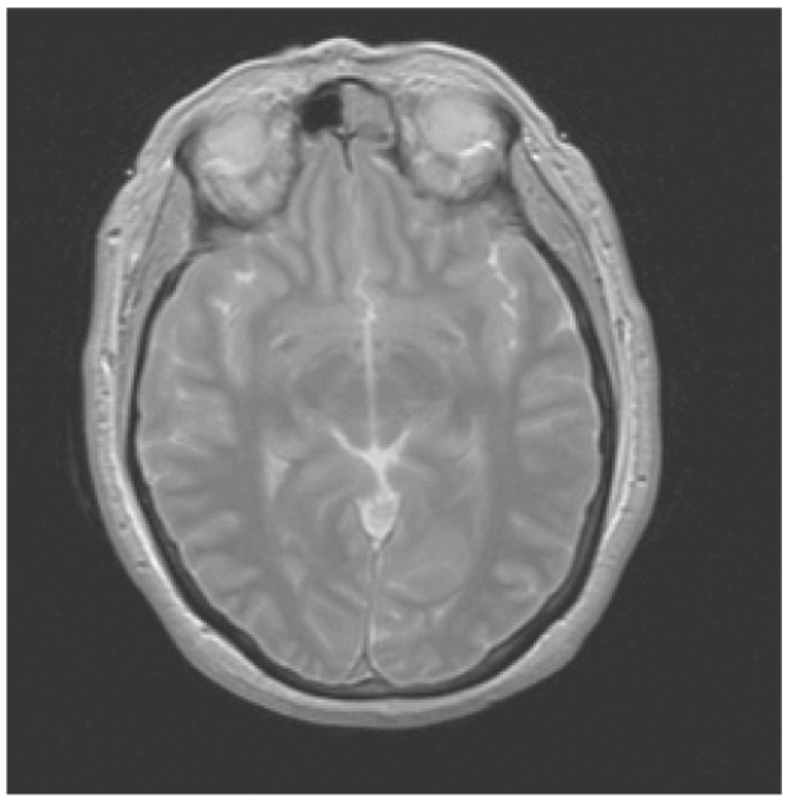
P_D-weighted — T_2-weighted — T_1-weighted

Typical Brain Tissue Parameters Measured at 1.5 T

Tissue Type	Relative P_D		T_2 (ms)		T_1 (ms)	
White matter	0.61		67		510	
Gray matter	0.69	brighter	77	brighter	760	brighter
Cerebrospinal fluid CSF	1.00		280		2650	

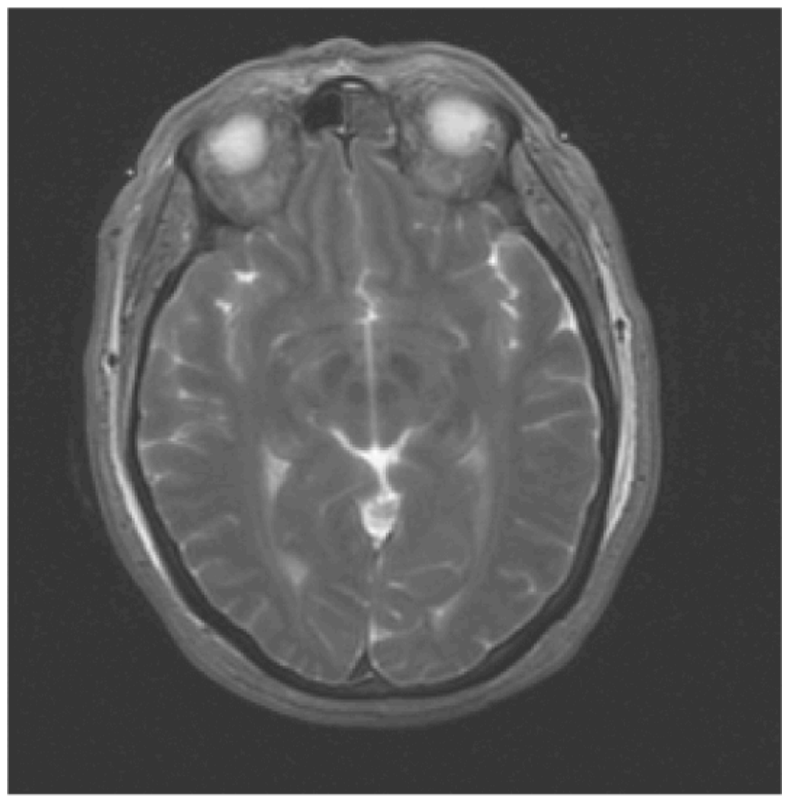
Proton Density (P_D) - weighted contrast

- Intensity proportional to number of hydrogen nuclei.
- Long T_R to allow tissues to be at equilibrium (largest possible longitudinal magnetization vector available to flip).
- Short T_E to avoid signal loss due to dephasing.
- Offers highest signal-to-noise (no loss to T1 or T2 relaxation).



T_2 - weighted contrast

- Intensity greater for tissues with long T_2, e.g. water (CSF).
- Long T_R to allow tissues to be at equilibrium (largest possible longitudinal magnetization vector available to flip).
- Intermediate T_E to permit differentiation between tissues with short and long T_2.



T_1 - weighted contrast

- Intensity greater for tissues with short T_1.
- Intermediate T_R to differentiate between tissues with long and short T_1
- Short T_E to minimize differentiation of tissues due to T_2 and to avoid signal loss due to dephasing.